MACHAKOS UNIVERSITY

University Examinations for 2019/2020Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

FIFTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

EEE 546: INFORMATION THEORY AND CODING

DATE: 23/10/2020 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) The internal operation of the system (channel coding, modulation method, etc) can be optimized with respect to the properties of the used transmission medium in terms of the key resources. What are the two key resources? (2 marks)

b) Observing a random variable X that takes its values from the discrete set $\Omega_X = \{a_1, a_2 \;\; a_K \}$, write down an expression for the information obtained from a particular observation a_m. (2 marks)

c) Determine the entropy of the following soucres.

d) Throwing a fair dice whose six possible outcomes are equiprobable. The memoryless model of English text which results to the following probabilities or rates of occurrence of different alphabets:

Source Symbol a_i	Probability p_i
Space	0.186
A	0.064
B	0.013
C	0.022
D	0.032
E	0.103
F	0.021
G	0.015
H	0.047
I	0.058
J	0.001
K	0.005
L	0.032
M	0.020
N	0.057
O	0.063
P	0.015
Q	0.001
R	0.048
S	0.051
T	0.080
U	0.023
V	0.008
W	0.017
X	0.001
Y	0.016
Z	0.001

(6 marks)

e) Consider the following communications system model

From this model, explain what conditional entropy and mutual information are and determine expressions for the two based on the model. (6 marks)

f) In all error control methods, the idea is to create certain known "structure" or redundancy to the transmitted bit stream and thereon to the transmitted symbol stream Adding new (redundant) bits to the information bit stream. Then, if the modulation does not depend on channel coding, this means that also the amount of transmitted symbols increases (redundant symbols). This is the classical approach. As an alternative, channel coding and modulation can be designed jointly, and the redundancy can be built in to the transmitted signal without increasing the amount of symbols. This is called trellis coding. On the receiver side, this known structure (redundancy) can then be deployed for error control in data decoding stage, in three different ways. State these three ways and explain them briefly. (3 marks)

g) Explain what coding gain is. (3 marks)

h) State the Shannon-Hartley Theorem with an explanation of its significance. Show that, according to this theorem, arbitrarily low bit-error rates cannot be achieved from a digital channel if E_b/N_0 is less than about -1.6 dB where E_b is the average energy per bit and $N_0/2$ is the two-sided power spectral density of additive white Gaussian noise. (8 marks)

QUESTION TWO (20 MARKS)

a) Category 5 cable, commonly referred to as CAT-5, is a twisted pair cable for computer networks. Since 2001, the variant commonly in use is the Category 5e specification (CAT-5e). The cable standard provides performance of up to 100 MHz and is suitable for most varieties of Ethernet over twisted pair up to 1000BASE-T (Gigabit Ethernet). We would like to transmit information over a CAT-5 twisted pair cable at 500 megabits per second (Mbps).

   i. Is a signal-to-noise ratio (SNR) of 30 dB adequate to transmit 500 Mbps? (4 marks)

   ii. What is the SNR required to just adequately transmit at 500 Mbps? Express your answer in decibels for the SNR (that is, SNR_{dB}). (3 marks)

b) A channel has B = 4 KHz and a signal-to-noise ratio of 30 dB. Determine maximum information rate for 128-level encoding. (4 marks)

c) Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel, determine the channel capacity C. (3 marks)

d) Prove that the information measure is additive: that the information gained from observing the combination of N independent events, whose probabilities are pi for i = 1....N, is the sum of the information gained from observing each one of these events separately and in any order (3 marks)

e) Suppose that ravens are black with probability 0.6, that they are male with probability 0.5 and female with probability 0.5, but that male ravens are 3 times more likely to be black than are female ravens. If you see a non-black raven, what is the probability that it is male? How many bits worth of information are contained in a report that a non-black raven is male? Rank-order for this problem, from greatest to least, the following uncertainties:

   i. Uncertainty about colour;

   ii. Uncertainty about gender;

   iii. Uncertainty about colour, given only that a raven is male;

iv. Uncertainty about gender, given only that a raven is non-black. (3 marks)

QUESTION THREE (20 MARKS)

a) Define what Hamming distance. . (2 marks)

b) Given two bit words $\mathbf{c}_1 = [1, 0, 0, 0]$ and $\mathbf{c}_2 = [0, 1, 0, 1]$. Determine the Hamming distance between the two codes. Assuming binary modulation with alphabet $\{+a,-a\}$, the corresponding symbol words or signal vectors are $\mathbf{s}1 = [+a,-a,-a,-a]$ and $\mathbf{s}2 = [-a, +a,-a, +a]$; determine the Euclidian distance based on this and derive an expression relating the Hamming distance with the Euclidian distance based on the binary modulation assumption. (6 marks)

c) Consider a binary symmetric communication channel, whose input source is the alphabet X = {0, 1} with probabilities {0.5, 0.5}; whose output alphabet is Y = {0, 1}; and whose channel matrix is

$$\begin{pmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{pmatrix}$$

i. What is the entropy of the source, H(X)? (2 marks)

ii. What is the probability distribution of the outputs, p(Y), and the entropy of this output distribution, H(Y)? (2 marks)

iii. What is the joint probability distribution for the source and the output, p(X, Y), and what is the joint entropy, H(X, Y)? (2 marks)

iv. What is the mutual information of this channel, I(X; Y)? (2 marks)

v. How many values are there for ϵ for which the mutual information of this channel is maximal? What are those values, and what then is the capacity of such a channel in bits? (2 marks)

vi. For what value of ϵ is the capacity of this channel minimal? What is the channel capacity in that case? (2 marks)

QUESTION FOUR (20 MARKS)

a) Consider n different discrete random variables, named X1,X2, ...,Xn, each of which has entropy

$H(X_i)$. Suppose that random variable X_j has the smallest entropy, and that random variable X_k has the largest entropy. What is the upper bound on the joint entropy $H(X_1,X_2, ...,X_n)$ of all these random variables? Under what condition will this upper bound be reached? What is the lower bound on the joint entropy $H(X_1,X_2, ...,X_n)$ of all these random variables? Under what condition will the lower bound be reached? (6 marks)

b) Suppose that a continuous communication channel of bandwidth W Hertz and a high signal-to-noise ratio, which is perturbed by additive white Gaussian noise of constant power spectral density, has a channel capacity of C bits per second. Approximately how much would C be degraded if suddenly the added noise power became 8 times greater? (5 marks)

c) An error-correcting Hamming code uses a 7 bit block size in order to guarantee the detection, and hence the correction, of any single bit error in a 7 bit block. How many bits are used for error correction, and how many bits for useful data? If the probability of a single bit error within a block of 7 bits is p = 0.001, what is the probability of an error correction failure, and what event would cause this? (4 marks)

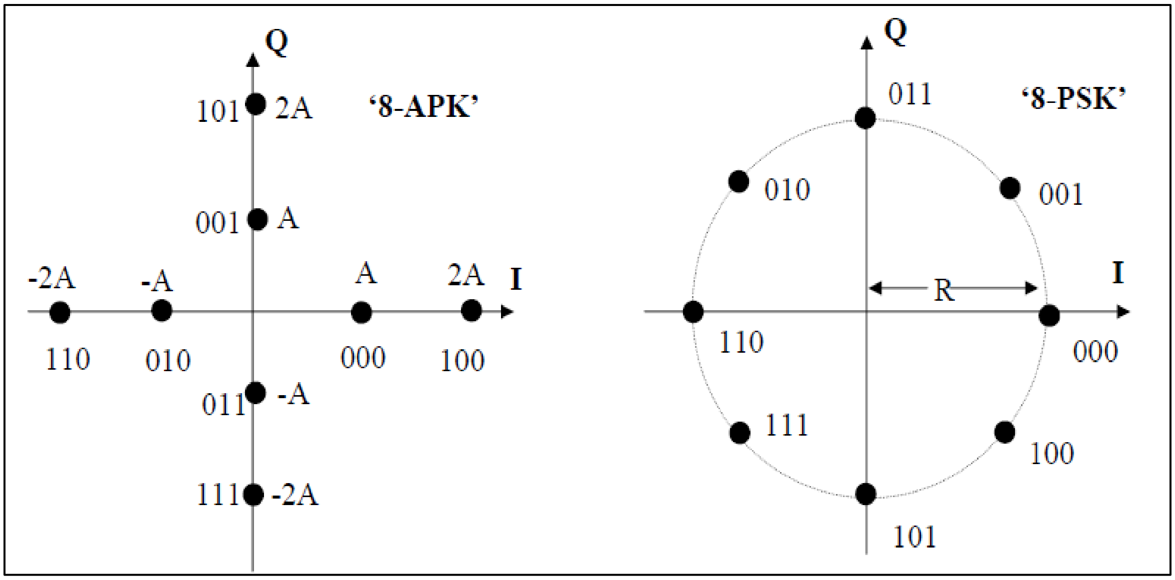
d) What is the shortest possible code length, in bits per average symbol, that could be achieved for a six-letter alphabet whose symbols have the following probability distribution? (5 marks)

$$\left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32} \right\}$$

QUESTION FIVE (20 MARKS)

a) Why is Gray coding used in multi-level digital modulation schemes? (2 marks)

b) The constellation diagrams for an '8-APK' modulator and an '8-PSK' modulator are shown below:



Assuming all symbols are equally likely, calculate the average energy per bit, E_b, for each of these modulation schemes when the symbol rate is 1 kbaud. Show that if $R \approx 1.58A$, the average energy per bit is the same for both schemes. (5 marks)

Symbol detection is achieved by choosing the nearest of the eight symbols on the constellation diagram to a received symbol and therefore the minimum distance between symbols is taken as a measure of noise immunity. Which of the two schemes has the greater noise immunity for a given E_b/N_0?

(3 marks)

Sketch the waveform produced by each of the schemes above when the 3 kb/s bit-stream is as follows:

0 0 0 1 1 0 0 1 1 1 0 1

and the carrier frequency is 2 kHz. The effect of band-limiting and pulse-shaping need not be shown. (5 marks)

c) Consider a (6,3) linear block-code that is generated by a generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Is the code systematic? Write down the check-bit equations and tabulate the code words and their weights. Find the parity check matrix **H** for this code. What is the minimum distance? How many errors can be:

i. Corrected

ii. Detected

with this code in hard decoding? How would you do the actual error correction detection? (5 marks)