MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SESCOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 307: FINANCIAL ECONOMICS

DATE: 14/4/2023 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (30 marks)

a) In a market in which the Arbitrage Pricing Theory (APT) model holds, the expected return is given;

$$E(R_i) = \lambda_0 + \lambda_1 b_{i,1} + \lambda_2 b_{i,2} + \ldots$$

i. Define all the terms in the equation. (2 marks)

Assume the risk – free rate $r_f = 0.04$. Consider two well diversified portfolios P_i with the following features in a two-factor model.

	P_1	P_2
$E(R_i)$	15.50%	11.95%
$b_{i,1}$	(a)	(b)
$b_{i,2}$	1.5	0.7

Determine the values (a) and (b) for $\lambda_1 = 0.05$ and $\lambda_2 = 0.06$ (3 marks)

b) i) State THREE forms of market efficiency under efficient market hypothesis (3 marks)

ii) State 2 types of CAPM risks. (2 marks)

c) An insurance company will be required to make a payout of £500 on a particular risk event, which is likely to occur with a probability of 0.4. The utility for any level of wealth, w, is given by:

$U(w) = 4{,}000 + 0.5w$

The insurer's initial level of wealth is £6000. Calculate the minimum premium the insurer will require in order to take on the risk. (5 marks)

d) Interest rates are fixed at either 4% pa with probability 0.2 or 5% pa with probability 0.8. What is the standard deviation of the present value of a payment of £25,000 in 5 years' time (5 marks)

e) Starting from the formula for the price of a call option, use put-call parity to derive the formula for the price of a put option (5 marks)

f) Consider the two risky assets, A and B, with cumulative probability distribution functions

$$F_A(w) = w$$

$$F_B(w) = w^{1/2}$$

In both cases, $0 \leq w \leq 1$.

Show that A is preferred to B on the basis of first-order stochastic dominance (5 marks)

QUESTION TWO (20 MARKS)

a) Outline the similarities and differences between deterministic and stochastic interest rate. (4 marks)

b) A stochastic interest rate model assumes that the interest rates in different years are independent and identically distributed normal random variables with mean 8% and standard deviation 2%. Calculate the mean and variance of the accumulated value, at time 2, of an initial investment of £10,000. (6 marks)

c) The annual rates of return on an investments funds are assumed to be independent and identically distributed. Each year the distribution of $(1 + i)$ is lognormal with parameters $\mu = 0.07$ and $\delta^2 = 0.006$ where i is the annual yield on the fund.

Calculate the amounts that should be invested in the fund immediately to ensure an accumulated value of at least £500,000 in ten years' time with probability of 0.99.

(10 marks)

QUESTION THREE (20 MARKS)

a) Verify that the Garman-Kohlhagen formula for a European call option on a dividend-paying share satisfies the boundary condition. (4 marks)

b) A forward contract is arranged where an investor agrees to buy a share at time T for an amount K. It is proposed that the fair price for this contract at time t is:

$$f(t, S_t) = S_t - Ke^{-r(T-t)}$$

Show that this:

i. satisfies the boundary condition (2 marks)

ii. satisfies the Black-Scholes PDE. (6 marks)

c) An investor buys, for a premium of 187.06, a call option on a non-dividend-paying stock whose current price is 5,000. The strike price of the call is 5,250 and the time to expiry is 6 months. The risk-free rate of return is 5% *pa* continuously compounded. The Black-Scholes formula for the price of a call option on a non-dividend-paying share is assumed to hold.

i. Calculate the price of a put option with the same time to maturity and strike price as the call. (2 marks)

ii. The investor buys a put option with strike price 4,750 with the same time to maturity.

Calculate the price of the put option if the implied volatility were the same as that in (i). (You need to estimate the implied volatility to within 1% *pa* of the correct value.)

(6 marks)

QUESTION FOUR (20 MARKS)

a) Explain the four axioms that are required to derive the expected utility theorem (8 marks)

b) An investor can invest in two assets, A and B:

	A	B
expected return	6%	8%
variance	4%%	25%%

The correlation coefficient of the rate of return of the two assets is denoted by ρ and is assumed to take the value 0.5.
The investor is assumed to have an expected utility function of the form

$$E\alpha(U) = E(r_p) - \alpha\, Var(r_p)$$

where α is a positive constant and r_p is the rate of return on the assets held by the investor.

(i) Determine, as a function of α , the portfolio that maximises the investor's expected utility. (10 marks)

(ii) Show that, as α increases, the investor selects an increasing proportion of Asset A. (2 marks)

QUESTION FIVE (20 MARKS)

a) Define in the context of mean- variance portfolio
   i. an inefficient portfolio
   ii. an efficient portfolio (2 marks)

b) State the two assumptions about investor behaviour that are needed for the existence of efficient portfolios (2 marks)

An investment universe includes two assets A and B, with expected return on asset I of r_i and variance V_i as set out below:

Asset I	**Expected return r_i**	**Variance of return V_i**
A	$r_A = 0.05$	$V_A = 0.16$
B	$r_B = 0.07$	$V_B = 0.25$

The correlation of return is $C_{AB} = - 0.2$

In an efficient portfolio, let **a** be the proportion which is held in Asset A.

c) Express the portfolio variance V in terms of quadratic function in **a**, showing your workings. (5 marks)

Let R be the expected return in the portfolio.

d) Express the portfolio variance V in terms of a quadratic function in R, using your result from part (c) and showing your workings. (Your expression should not include **a**) (5 marks)

The expression in part (d) represents the efficient frontier. An investor uses a utility function that gives rise to an indifference curve;

$$V = 16R – 200R^2$$

e) Determine the two portfolios on the efficient frontier that also lie on the investor's indifference curve. (4 marks)

f) Comment on the implications for part (e) if short selling is not allowed in the market. (2 marks)