**MACHAKOS UNIVERSITY**

**ISO 9001:2008 Certified**

**UNIVERSITY SUPPLEMENTARY EXAMINATIONS**

**SMA 401: TOPOLOGY II**

**DATE: …………………………….. TIME 2 HOURS**

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**INSTRUCTIONS TO CANDIDATES**

**Answer ALL the questions in Section A and ANY TWO Questions in Section B**

**SECTION A**

**QUESTION ONE (30 MARKS) COMPULSORY**

a) Define the following terms
   i) A component
   ii) Connected sets
   iii) Disconnected sets
   iv) Homeomorphism
   v) Locally compact sets (5 marks)

b) Let $S \cap T$ be a disconnection of A. show that $S \cap A$ and $T \cap A$ are separated. (5 marks)

c) Prove that the open interval $B = (0, \frac{1}{8})$ on the real line $\mathbb{R}$ with the usual topology is not compact. (5 marks)

d) Consider the following topology on $X = \{1,2,3,4,5\}$ $\rho = \{X, \emptyset, \{1,2,3\}, \{3,4,5\}, \{3\}\}$. Now $A = \{1,4,5\}$. Show that A is disconnected. (5 marks)

e) Prove that a set is disconnected if and only if it is not a union of two non-empty separated sets.
. (5 marks)

f) Prove that every compact subset of a Housdorff space is closed. (5 marks)

**SECTION B**

**QUESTION TWO 20 MARKS**

a) Show that if A and B are non-empty separated sets. Then, $A \cup B$ is disconnected (5 marks)

b) Show that every metric space is hausdorff space (5 marks)

c) Prove that if $F$ is closed subset of a compact space X, then F is also compact. (5 marks)

g) Prove that if Y is sequentially compact then it is countably compact. (5 marks)

**QUESTION THREE 20 MARKS**

a) Show that an infinite subset A of a discrete space X is not compact (7 marks)

b) Prove that a closed subset F of a compact set X is also compact. (7 marks)

c) Prove that a topological space X is compact if and only if $\{F_i\}$ of closed subsets of X satisfies the finite intersection property. (6 marks)

**QUESTION FOUR 20 MARKS**

a) Define hereditary as used in topological spaces (2 marks)

b) $T_{0\ and}\ T_1$ spaces are hereditary prove (6 marks)

c) Let $(X, \rho)$ be a topological space $(X, \rho)$ is a $T_1$ space iff each singleton subset $\{x\}$ is closed in $(X, \rho)$ (6 marks)

d) Prove that if $X, \rho)$is a topological space which is $T_2$. Then every convergent sequence of points of X has a unique limit. (6 marks)

**QUESTION FIVE 20 MARKS**

a) Prove that if A and B are disjoint compact subsets of Hausdorff spaces X. Then ∃ open set G and H such that $A \subset G$ and $B \subset H$ and $G \cap H = \emptyset$ (6 marks)

b) Let $\mathbb{Z}$ be the set of integers. Is it sequentially compact? (3 marks)

c) Let $G \cup H$ be a disconnection of A and B be a connected subset of A. Show that $B \cap G = \emptyset$ or $B \cap H = \emptyset$. ( 6 marks)

d) Prove that if Y is compact then it is countably compact. (5 marks)