MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCENCE

THIRD YEARSPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF ECONOMICS AND STATISTICS
BACHELOR OF EDUCATION
BACHELOR OF ARTS

SMA 460: STOCHASTIC PROCESSES

DATE: 26/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given the difference, differential equations for a pure birth process is:

$$P_n^{'}(t) = -\lambda_n P_n(t) + \lambda_{n-1} P_{n-1}(t)\ , \quad n \geq 1$$

and

$$P_0^{'}(t) = -\lambda_0 P_0(t), \quad n = 0$$

Write down the difference differential equations when:

i. $\lambda_n = n\lambda$ (2 marks)

ii. $\lambda_n = \lambda\left[\dfrac{1+an}{1+\lambda at}\right]$ (2 marks)

b) Define the following terms as used in Markov chains:

i. Persistent state.
ii. Transient state
iii. Ergodic markov chain (6 marks)

c) Let X have the distribution of the geometric form given by

$$P_k = \Pr\{X = k\} = q^k p \,,\; k = 0,1,2,\ldots and \;\; q = 1 - p$$

i. Find the probability generating function of X. (3 marks)

ii. Hence determine the mean and the variance of X. (5 marks)

d) Let X and Y be two independent random variables with probability generating functions (P.g.f) $P_X(s)$ *and* $P_Y(s)$ respectively. Find the p.g.f of the random variable $Z = X - Y$

(4 marks)

e) A markov chain whose states are has the following transition probability matrix. Classify the states.

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

(8 marks)

QUESTION TWO

a) Given the difference, differential equations for a Poisson process are:

$$P_n^{'}(t) = -\lambda P_n(t) + \lambda P_{n-1}(t) \;,\quad n \geq 1$$

$$and$$

$$P_0^{'}(t) = -\lambda P_0(t) \,,\qquad n = 0$$

Use Feller's method to determine the mean and variance of this process, given the initial conditions are $P_n(0) = 1$ for $n = 0$ and zero otherwise. (11 marks)

b) Consider the two state Markov chain:

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0 < a, b < 1$$

Obtain the stationary distribution. (9 marks)

QUESTION THREE

a) Define the following terms :

i. Stochastic matrix. (2 marks)

ii. An irreducible markov chain (3 marks)

b) Classify the states of the following Markov chain.

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(15 marks)

QUESTION FOUR

a) Consider the difference, differential equations of the Yule-furry process i.e:

$$P_n^{'}(t) = -n\lambda P_n(t) + (n-1)\lambda P_{n-1}(t) \ , \quad n \geq 1$$
$$and$$
$$P_0^{'}(t) = 0, \qquad n = 0$$

Given the initial conditions are $P_n(0) = 1$ when $n = 1$ and zero otherwise.

Show by the generating function method, that the solution of the equation is given by

$P_n(t) = e^{-\lambda t}(1 - e^{-\lambda t})^{n-1}, \ n \geq 1$ (14 marks)

b) Using probability generating function of the distribution in (a) show that the mean and variance are $e^{\lambda t}$ *and* $e^{\lambda t}(e^{\lambda t} - 1)$ respectively. (6 marks)

QUESTION FIVE

a) Let $\{X_n : n = 1, 2, 3, ...\}$ be a stochastic process with probability distribution given by

$$\Pr\{X_n = k\} = \begin{cases} p\,q^{k-1}, & k = 1, 2, 3, \ldots and \ q = 1 - p \\ 0, & otherwise \end{cases}$$

Find the probability generating function of $\{X_n\}$. Hence obtain the mean and the variance of the process. (8 marks)

b) The joint distribution of two random variables X and Y is given by

$$P_{jk} = \Pr\{X = j, Y = k\} = \begin{cases} p^2 q^{j+k}, & j, k = 0, 1, 2, 3, \ldots and \ q = 1 - p \\ 0, & otherwise \end{cases}$$

Obtain

i. the bivariate probability generating function of X and Y. (3 marks)

ii. the Covariance of X and Y. (9 marks)