



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 202: MODELS IN ACTUARIAL SCIENCE

DATE: 11/4/2023

TIME: 2:00 – 4:00 PM

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)(30 MARKS)

- a) A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract.
Basis: Mortality: AM92 Select
Interest: 4% per annum (5 marks)
- b) A life insurance company is setting the pricing bases for some of its products. Discuss the importance of the assumptions for the investment return, expenses and withdrawal rates, when setting the pricing bases for the following products a without profit term insurance. (3 marks)
- c) A woman deposits £200 in a special bank account. Interest is paid to the woman every year on her birthday for five years. The capital is returned after exactly five years, along with any interest accrued since her last birthday. Interest is calculated at an effective rate of 6% *pa*.
Calculate the present value of the interest received by the woman (3 marks)
- d) A payment of €2,000 is due in 4 years' time. Calculate the present value using:
i. simple interest of 3% *pa* (2 marks)
ii. simple discount of 3% *pa* (2 marks)
i. compound interest of 3% *pa*. (2 marks)

- e) The force of interest at time t , where $0 \leq t \leq 10$, is given by $\delta(t) = 0.07$.
Calculate the accumulated value at time 10 of a payment stream, paid continuously from time 5 to time 10, under which the rate of payment at time t is $10e^{0.05t}$.
(5 marks)
- f) A price P is paid (by the investor) in return for a series of interest payments of D payable at the end of each of the next n years, and a final redemption payment of R payable at the end of the n years. The investor earns an annual effective rate of return of i .
Calculate n , given that $P = 83.73$, $D = 4$, $R = 101$ and $i = 6\%$ (4 marks)
- g) Calculate the value of $a_{40:\overline{3}|}$ when the effective annual rate of interest is 6% and:
 $l_x = 100 - x$ at all ages $x \leq 100$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Calculate $\ddot{a}_{60:10|}$
Basis: Mortality: AM92 Ultimate
Interest: 6% *pa* effective (5 marks)
- b) A continuously payable temporary annuity is sold to a life aged exactly 40. The annuity makes payments at a rate of 5,000 *pa* until age 60 or the policyholder's earlier death. Calculate the expected present value of the annuity payments, using AM92 Select mortality and an interest rate of 4% *pa* effective (5 marks)
- c) An annuity is payable continuously throughout the lifetime of a person now aged exactly 60, but for at most 10 years. The rate of payment at all times t during the first 5 years is £10,000 *pa*, and thereafter it is £12,000 *pa*.
The force of mortality of this life is 0.03 *pa* between the ages of 60 and 65, and 0.04 *pa* between the ages of 65 and 70.
Calculate the expected present value of this annuity assuming a force of interest of 0.05 *pa*.
(10 marks)

QUESTION THREE (20 MARKS)

- a) A 182-day treasury bill, redeemable at \$100, was purchased for \$96.50 at the time of issue and later sold to another investor for \$98 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.
- Calculate the length of time in days for which the initial purchaser held the bill. (2 marks)
 - Calculate the annual simple rate of return achieved by the second investor. (2 marks)
 - Calculate the annual effective rate of return achieved by the second investor. (2 marks)
- b) Calculate the effective annual rate of interest for:
- a transaction in which £200 is invested for 18 months to give £350. (2 marks)
 - transaction in which £100 is invested for 24 months and another £100 is invested for 12 months (starting 12 months after the first investment) to give a total of £350. (2 marks)
- c) The force of interest at time t is given by:
- $$\delta(t) = 0.01t + 0.04 \quad 0 \leq t \leq 5$$
- Calculate the present value at time 0 of a payment stream, received continuously from time 0 to time 5, under which the rate of payment at time t is $0.5t + 2$. (10 marks)

QUESTION FOUR (20 MARKS)

- a) A life insurance company adopts a strict underwriting approach under a particular class of business.
- Explain why it may also wish to use reinsurance to manage the mortality risk under the business. (4 marks)
 - Explain why, having entered into a reinsurance treaty, it would still wish to underwrite the business (4 marks)
- b) At a certain company, the probability of each employee leaving during any given year is 5%, independent of the other employees. Those who remain with the company for 25 years are given \$1,000,000. What is the expected present value of this payment to a new starter, assuming an interest rate of 7% pa and ignoring the possibility of death? (3 marks)

- c) Claire, aged exactly 30, buys a whole life assurance with a sum assured of £50,000 payable at the end of the year of her death. Calculate the standard deviation of the present value of this benefit using AM92 Ultimate mortality and 6% pa interest. (4 marks)
- d) Explain how an insurance company uses risk classifications to control the profitability of its life business. (4 marks)

QUESTION FIVE (20 MARKS)

- a) An impaired life aged 40 experiences 5 times the force of mortality of a life of the same age subject to standard mortality. A two-year term assurance policy is sold to this impaired life, and another two-year term assurance is sold to a standard life aged 40. Both policies have a sum assured of £10,000 payable at the end of the year of death. Calculate the expected present value of the benefits payable to each life assuming that standard mortality is AM92 Ultimate and interest is 4% pa. (10 marks)
- b) Assuming that the force of mortality between consecutive integer ages is constant in the AM92 Ultimate table, calculate the exact value of $\bar{A}_{50:2}$ using a rate of interest of 4% pa. (10 marks)