MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMISTS II

DATE: 4/12/2019 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

- **(i)** **Answer question one (Compulsory) and any other two questions**
- **(ii)** **Do not write on the question paper**
- **(iii)** **Show your workings clearly**

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate by means of integration by substitution:

$\int 6x^2\,(2x^3+2)^{99}dx$ (3 marks)

b) The input-output ratio coefficients for a three sector economy are given as:

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{23} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}$ while the final demand matrix is given as: $D = \begin{bmatrix} 10 \\ 5 \\ 6 \end{bmatrix}$

Determine the total output required in each of the three sectors to meet the inter-industry input requirements and the final demand. (7 marks)

c) Given the following production function with two inputs, Labour (L) and Capital (K):

$Q = A K^{\alpha} L^{\beta}$

i) Derive Euler's Theorem (5 marks)

ii) If the function is now given as:

$Q = A L^{\frac{1}{2}} K^{\frac{3}{4}}$

Prove Euler's Theorem (4 marks)

d) The production function for Fix it garage which services cars and trucks is given as:

$Q = 15L^{2/3}K^{1/3}$

Where, L and K are the units of Labour and Capital used, respectively. If each unit of labour used costs $5 while each unit of capital cost $3:

i) Find the units of labour and capital to be used so as to maximize the output of the garage given that the garage has only $450 dollars at its disposal. (5 marks)

ii) Do the optimal values of K and L computed in i) above maximize the output of the garage? (6 marks)

QUESTION TWO (20 MARKS)

a) Given the following matrix:

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 8 & 9 & 2 \\ 1 & 5 & 3 \end{bmatrix}$$

Compute: (6 marks)

i) Its determinant

ii) Its transpose

b) Given the following information:

$$Qdx = 20 - 3Px + 4Py + 0.2Y$$

$$Px = 1 \quad Py = 2 \quad M = 500$$

Where: Qdx is the quantity demanded for good X.

Px is the price for good X.

Py is the price for good Y.

M is the consumer's income.

i) Under what category of goods do good X and good Y fall and why? (4 marks)

ii) Determine whether these 2 goods are substitutes or complements. (6 marks)

c) If we assume we have only two factors of production, capital and labour, we can denote the production function as:

$Q = f(L, K)$

Given the production function, compute the marginal rate of technical substitution (MRTS) of capital and labour. (4 marks)

QUESTION THREE (20 MARKS)

a) Given these two matrices

$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 6 \\ 4 & 0 & 8 \end{bmatrix}$

Compute AB and BA and comment on your answers. (4 marks)

b) Compute the derivatives of the following functions: (10 marks)

i. $y = \ln(3x^3 - 2x^2)$

ii. $y = 7^{4y^2-6}$

iii. $y = log_a(3x^2 + 7x)$

iv. $\frac{1+x^3}{(x+5)(3x-2)}$

v. $Y = e^X 4lnX^2$

c) A utility function is given as:

U=Q½ Q4/3

i) Determine the marginal utility of good 1 and 2 (3 marks)

ii) Determine the nature of marginal utility for each of the goods. (3 marks)

QUESTION FOUR (20 MARKS)

a) Get the first order, second order and cross partial derivatives of the following functions. From your solutions, prove Young's Theorem. (5 marks)

i) $Y = 8X_1 + 15X_1^2 X_2 + 3X_2^3$

ii) $Y = 2X^2 + 3XW + 7W^3$

b) Determine the nature of returns to scale for the following functions: (6 marks)

i) $Q = 80K^{0.2}L^{0.5}$

ii) $Q = A L^{\frac{1}{2}} K^{\frac{3}{4}}$

c) A three product market model is given as:

$$Q_{d1} = 50 - 2P_1 + 5P_2 - 3P_3 \qquad Q_{s1} = 8P_1 - 5$$

$$Q_{d2} = 22 + 7P_1 - 2P_2 + 5P_3 \qquad Q_{s2} = 12P_2 - 5$$

$$Q_{d3} = 17 + P_1 + 5P_2 - 3P_3 \qquad Q_{s3} = 4P_3 - 1$$

Using Cramer's rule, compute the equilibrium quantities and prices for the three products.

(9 marks)

QUESTION FIVE (20 MARKS)

a) The supply and demand functions for fruits are given as:

$$P = -10 + Q$$

$$P = 40 - 2Q$$

i) Determine the producer surplus when the Price = 30 (5 marks)

ii) Determine the consumer surplus when the Price = 10 (5 marks)

b) The total cost function of a firm that produces two goods (Q_1 and Q_2) is given as:

$$C = 8Q_1^3 + 6Q_2^2 + 4Q_1Q_2 - 10$$

Required:

i) Compute the marginal costs of the two goods. (3 marks)

ii) Determine the nature of the marginal costs computed above (4 marks)

c) Get the partial derivatives of the following function:

$Y = f(X_1 X_2) = [18X_1^2 + 3X_2 X_1 - X_2^2]^4$ (3 marks)