MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF EDUCATION (SCIENCE)

SMA 464: DESIGN AND ANALYSIS OF EXPERIMENTS I

DATE: 23RD JANUARY, 2025 **TIME: 11.00 AM – 1.00 PM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Explain the following terms
   i. Randomization **(2 marks)**
   ii. Replication **(2 marks)**
   iii. Blocking **(2 marks)**

b) Describe the guidelines for designing experiments **(7 marks)**

c) Illustrate the analysis of the single factor fixed effects model **(5 marks)**

d) Explain the use of contrasts in multiple comparison methods **(4 marks)**

e) A manufacturer suspects that the batches of raw material furnished by his supplier differ significantly in calcium content. There are four batches currently in the warehouse. A chemist makes four determinations on each batch and obtains the following data:

Batch 1	Batch 2	Batch 3	Batch 4
3.6	3.9	3.5	3.8
3.8	3.6	3.6	3.0
3.5	3.2	3.4	3.7
3.4	3.5	3.9	3.9

Is there significant variation in calcium content from batch to batch? Use $\alpha = 0.05$.
(8 marks)

QUESTION TWO (20 MARKS)

a) Describe the following types of experimental designs

i. Randomized complete block design **(2 marks)**

ii. Latin square design **(2 marks)**

b) An experiment was conducted in which a shape factor was determined for several different nozzle designs at six levels of jet efflux velocity. Interest focused on potential differences between nozzle designs, with velocity considered as a nuisance variable. The data are shown below:

Nozzle design	Jet efflux velocity (m/s)				
	11.73	14.37	16.59	20.43	23.46
1	0.8	0.8	0.8	0.8	0.8
2	0.9	0.9	0.9	0.9	0.8
3	0.9	0.9	1.0	0.9	0.9
4	1.1	1.0	1.0	0.9	0.9

Does nozzle design affect the shape factor? **(8 marks)**

c) The effect of five different ingredients (*A, B, C, D, E*) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately 4.5 hours, so only five runs can be made in one day. The experimenter decides to run the experiment as a Latin square so that day and batch effects may be systematically controlled. She obtains the data that follow. Analyse the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Day	Batch				
	1	2	3	4	5
1	A=8	B=7	D=1	C=7	E=3
2	C=11	E=2	A=7	D=3	B=8
3	B=4	A=9	C=10	E=1	D=5
4	D=6	C=8	E=6	B=6	A=10
5	E=4	D=2	B=3	A=8	C=8

(8 marks)

QUESTION THREE (20 MARKS)

a) Describe the following types of experimental designs

i. Balanced incomplete block design **(2 marks)**

ii. Graeco-Latin square design **(2 marks)**

b) Seven different hardwood concentrations are being studied to determine their effect on the strength of the paper produced. However, the pilot plant can only produce three runs each day. As days may differ, the analyst uses the balanced incomplete block design that follows. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Hardwood Concentration (%)	Days						
	1	2	3	4	5	6	7
2	14				20		17
4	26	20				19	
6		37	17				34
8	41		29	49			
10		45		50	43		
12			20		18	23	
14				36		30	27

(8 marks)

c) The yield of a chemical process was measured using five batches of raw material, five acid concentrations, five standing times (*A, B, C, D, E*), and five catalyst concentrations ($\alpha, \beta, \gamma, \delta, \varepsilon$). The Graeco-Latin square design that follows was used. Analyse the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Batch	Acid Concentration				
	1	2	3	4	5
1	$A\alpha = 26$	$B\beta = 16$	$C\gamma = 19$	$D\delta = 16$	$E\varepsilon = 13$
2	$B\gamma = 18$	$C\delta = 21$	$D\varepsilon = 18$	$E\alpha = 11$	$A\beta = 21$
3	$C\varepsilon = 20$	$D\alpha = 12$	$E\beta = 16$	$A\gamma = 25$	$B\delta = 13$
4	$D\beta = 15$	$E\gamma = 15$	$A\delta = 22$	$B\varepsilon = 14$	$C\alpha = 17$
5	$E\delta = 10$	$A\varepsilon = 24$	$B\alpha = 17$	$C\beta = 17$	$D\gamma = 14$

(8 marks)

QUESTION FOUR (20 MARKS)

a) Describe the following methods of performing posthoc ANOVA analyses

i. Scheffé's Method **(2 marks)**

ii. Fisher's least significant difference (LSD) **(2 marks)**

iii. Tukey's test **(2 marks)**

b) Use the Kruskal–Wallis test for the experiment in Q1 (e). Are the results comparable to those found by the usual analysis of variance? **(5 marks)**

c) Suppose that the batches for the experiment in Q1 (e) were selected at random from a population of batches, estimate the components of variance. **(3 marks)**

d) Write R code to perform analyses for the data in the following questions

i. Q1 (e) **(2 marks)**

ii. Q2 (b) **(2 marks)**

iii. Q4 (c) **(2 marks)**

QUESTION FIVE (20 MARKS)

a) A mechanical engineer is studying the thrust force developed by a drill press. She suspects that the drilling speed and the feed rate of the material are the most important factors. She selects four feed rates and uses a high and low drill speed chosen to represent the extreme operating conditions. She obtains the following results.

Drill speed	Feed rate			
	0.015	0.030	0.045	0.060
125	2.8	2.4	2.6	2.8
	2.7	2.5	2.7	2.9
200	2.8	2.9	3.0	2.9
	2.9	2.8	2.9	3.1

Analyze the data and draw conclusions. Use $\alpha = 0.05$. **(10 marks)**

b) An analyst wants to study whether the income levels of employees differ between three different training programs while controlling for the effect of years of experience. The data is shown below:

Employee	Training Program	Income (KES 1000)	Years of Experience
1	A	55	5
2	A	65	7
3	A	70	6
4	B	80	8
5	B	58	6
6	B	75	9
7	C	64	8
8	C	82	9
9	C	76	7

Analyze the data and draw conclusions. Use $\alpha = 0.05$. **(10 marks)**