



MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 104: ENGINEERING MATHEMATICS I

DATE: 25/1/2022

TIME: 11:00 – 1:00 PM

INSTRUCTIONS: Answer Question one (compulsory) any other two Questions

QUESTION ONE (30 MARKS)

- a) Convert the following cartesian and polar coordinates to the form stated
- i] $(-2.4, -3.6)$
- ii] $(3.6, 2.5 \text{ rad})$ (2 marks)
- b) If $(3i - j + mk)$ and $(-3i + 3j + 4k)$ are two orthogonal vectors, find the value of m (3 marks)
- c) If $\vec{a} = 3i - j - 4k$; $\vec{b} = -2i + 4j - 3k$ and $\vec{c} = i + 2j - k$. Find the projection of $(\vec{a} + 2\vec{b})$ along $\vec{c}$ (4 marks)
- d) Find the equation of a straight line passing through the point $(3, 4)$ such that the sum of its intercepts on the axes is 14 (4 marks)
- e) Find the equation of the parabola whose vertex is $(1, 1)$ and focus is $(4, 1)$ (4 marks)
- f) A focus of a standard ellipse is $(3, 0)$ and its eccentricity is $\frac{3}{5}$. Find its equation and the length of the latus rectum. (4 marks)
- g) Find the lengths of axes, eccentricity and foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (4 marks)

- h) If $\vec{a} = 2i + 5j + 3k$; $\vec{b} = 3i + 3j + 6k$ and $\vec{c} = 2i + 7j + 4k$. Find
- i] $(\vec{a} - \vec{b}) \times (\vec{c} - \vec{a})$ (3 marks)
- ii] $[(\vec{a} - \vec{b}) \times (\vec{c} - \vec{a})]$ (2 marks)

QUESTION TWO (20 MARKS)

- a) If $\vec{a} = 2i + 3j$, $\vec{b} = 7i - 2j$, find
- i] $|\vec{b} - \vec{a}|$ (1 mark)
- ii] Unit vector along $\vec{b} - \vec{a}$ (2 marks)
- b) Find the area of a parallelogram whose adjacent sides are $i + j - k$ and $2i - j + k$ (3 marks)
- c) A particle is displaced from point $(2, -3, 1)$ to $B(4, 2, 1)$ under the action of the constant forces $\vec{F}_1 = 12i - 5j + 6k$; $\vec{F}_2 = i + 2j - 2k$ and $\vec{F}_3 = 2i + 8j + k$. Find the work done by the forces on the particle (4 marks)
- d) Find the vector moment of three forces $(i + 2j - 3k)$; $(2i + 3j + 4k)$ and $(-i - j + 3k)$ acting on a particle at a point $P(0, 1, 2)$, about the point $A(1, -2, 0)$ (4 marks)
- e) Find the equation of the circle passing through $(5, 3)$, $(1, 5)$, and $(3, -1)$ (6 marks)

QUESTION THREE (20 MARKS)

- a) Find the area of a triangle determined by the vectors $\vec{a} = 3i + 4j$ and $\vec{b} = -5i + 7j$ (3 marks)
- b) Find the length of the tangent from the point $(6, -1)$ to the circle $x^2 + y^2 + 2x - 4y + 11 = 0$ (4 marks)
- c) Find the equation of the circle whose centre is the same as the centre of the circle $x^2 + y^2 - 6x + 2y + 4 = 0$ and which passes through $(7, 4)$ (4 marks)
- d) Show that the circle $x^2 + y^2 - 5x + 6y + 15 = 0$ and $2x^2 + 2y^2 - 4x + 12y + 12 = 0$ touch each other (4 marks)
- e) If $\vec{a} = i - 2j + 3k$; $\vec{b} = 2i + j + k$, find
- i) The unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ (3 marks)
- ii) Sine of the angle between the vectors $\vec{a}$ and $\vec{b}$ (2 marks)

QUESTION FOUR (20 MARKS)

- a) Find the equation of the parabola with vertex at (2,5) and directrix is $x = 3$. (4 marks)
- b) Find the lengths of the axes, eccentricity and foci of the hyperbole $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (4 marks)
- c) Find the equation of the ellipse given that $(-2,2)$ and $(3,-1)$ are on the ellipse (4 marks)
- d) Find the equation of the hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ given that the distance between the foci is 10 and the length of the latus rectum is $\frac{9}{2}$ (4 marks)
- e) Find the vertex, focus, equation of the axis and equation of the diretrix of the parabola $y^2 - 8x - 6y - 31 = 0$ (4 marks)

QUESTION FIVE (20 MARKS)

- a) Find the equation of the parabola with latus rectum joining the points $(4,5)$ and $(12,5)$ (10 marks)
- b) Find the centre, foci, equations of the directrices and length of the latus rectum of the ellipse $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ (10 marks)