MACHAKOS UNIVERSITY

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN (ELECTRICAL, CIVIL, MECHANICAL) ENGINEERING

ECU 201: ENGINEERING MATHEMATICS VI

DATE : TIME: 2 HOURS

INSTRUCTIONS: Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a. Define the following terms as used in probability:-

  i. Independent events (2 marks)

  ii. Axiomatic approach (2 marks)

b. Differentiate between primary data and secondary data giving examples of each. (2 marks)

c. Let P (A) =0.8, P (B) =0.7. Find P (AUB) if A and B are:

  i. Independent (1 mark)

  ii. Mutually exclusive events (2 marks)

d. Find the mean and standard deviation of the following probability distribution. (4 marks)

x	0	1	2	3	4	5
P(x)	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125

e. Obtain the Generating Function of $\{a_k\}$ given that $a_k = 1$ for all $k = 0, 1, 2...$

(3 marks)

f. The heights of adult females are normally distributed with mean of 63.6 inches and a standard deviation of 2.5 inches. Find the probability that;

i. A randomly selected female is 60 inches or shorter. (3 marks)

ii. A randomly selected female is between 62 inches and 67 inches tall.

(3 marks)

g. Here are the number of hours that 10 students spent working at their jobs the week before a final exam, and their scores on that exam. Calculate the correlation coefficient r and coefficient of determination for these data. (5 marks)

Hours	20	15	0	30	20	15	0	30	0	0
Score	87	71	89	60	86	89	91	65	94	92

h. A paint manufacturer wants to determine the average drying time of a new interior wall paint. If for 12 tests areas of equal size, he obtained a mean drying time of 66.3 minutes and a standard deviation of 8.4 minutes. Construct a 95% confidence interval for the true mean. (3 marks)

QUESTION TWO (20 MARKS)

a. The scores of five randomly selected students on test 1 and test 2 in a math class are as below. Find the equation of the regression line, treating the score on test 1 as x and the score on test 2 as y, and use it to predict the score when $x = 90$

(6 marks)

Student	Test 1 Score	Test 2 Score
1	83	82
2	86	84
3	76	63
4	92	83
5	71	55

b. Show that the sample mean is an unbiased estimator of the population mean that is $E\left[\bar{x}\right]=\mu$ (4 marks)

c. An experiment was conducted to investigate the viability of equipment designed to measure the foreign of an audio source. Three independent measurements were recorded by these equipment, they were 4.1, 5.2 and 10.2. Estimate the variance with 90% confidence interval. (5 marks)

d. Suppose you plan to survey employees to find their medical expenses, you want to be 95% confident that the sample mean is within ± sh.500. A pilot study shows that the standard deviation is about sh.8000. What sample size will you use? (3 marks)

e. On a typical day, a credit union opens 5 new accounts. Find the probability that the credit union opens exactly 6 accounts in the next two days. (2 marks)

QUESTION THREE (20 MARKS)

a. Define the following:

i. Classical approach (2 marks)

ii. Conditional probability (2 marks)

b. Given the data below;

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	15	18	23	30	13	8	6

i. Represent the data in an ogive curve ('less than' & 'more than') (6 marks)

ii. Estimate the Median from the graph (2 marks)

c. The probability that a person has O-Negative as his or her blood type is 0.06. If 15 people are selected at random, find the probability that at least one of them has O-Negative blood. (3 marks)

d. Find the following probabilities:

i. $P(-1.23 < z < 0.97)$ (3 marks)

ii. $P(z \geq 4.96)$ (2 marks)

QUESTION FOUR (20 MARKS)

a. For two events Q and R, $P(Q) = {}^{3}/_{4}$, $P(R) = {}^{3}/_{4}$ and $P(Q \cup R) = {}^{19}/_{20}$, compute:-

i. $P(Q \cap R)$ (3 marks)

ii. $P(Q/R)$ (2 marks)

b. Each of the three different students John, Hayley and Rita are given the same problem to solve. They work on the problem independently and have probability 0.8, 0.7 and 0.6 of solving it respectively.

i. What is the probability that none of them solve the problem? (2 marks)

ii. What is the probability that the problem will be solved by one or more of them? (3 marks)

c. Each of three different students, Joe, Hugh, and Rachael, are given the same problem to solve. They work on the problem independently and have probability 0.8, 0.7, and 0.6 of solving it, respectively.

i. What is the probability that none of them solve the problem? (2 marks)

ii. What is the probability that the problem will be solved (by one or more of them)? (3 marks)

iii. Granted the problem was solved, what is the probability that the solution is due to Rachael alone? (2 marks)

d. List any three continuous probability distributions. (3 marks)

QUESTION FIVE (20 MARKS)

a. Define the term Generating Function. (2 marks)

b. Show that the probability-generating function for the binomial distribution $p_j = \binom{n}{j} p^j q^{n-j}$ where $q = 1 - p$, is $p(x) = (q + px)^n$. Hence prove that the mean is $m = np$, and the variance is $\sigma^2 = npq$ (4 marks)

c. Find the generating function of $a_n = 5a_{n-1} - 6a_{n-2}, \; n \geq 2$ subject to the initial conditions : $a_0 = 1, a_1 = 3$ (7 marks)

d. Show that the integral of a probability generating function is equal to $E\left[\frac{1}{k+1}\right]$ (4 marks)

e. Let x be a random variable with p.g.f $P(s)$. Find the p.g.f of $Y = mX + n$. Where m,n are integers and $m \neq 0$. (3 marks)