



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 332: METHODS OF APPLIED MATHEMATICS

DATE:

TIME:

INSTRUCTION: Answer Question **ONE** which is compulsory and any other **TWO** Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) If 0.182 is the approximate value of $\frac{2}{11}$ find the absolute, relative and percentage errors. (4 marks)
- b) Construct the forward difference table, where $f(x) = \frac{1}{x}$, $x = 1(0.2)2, 4D$ (4 marks)
- c) For the following table of values, estimate $f(7.5)$, using Newton's backward difference interpolation formula (5 marks)
- d) If $f(x)$ is a polynomial of n^{th} degree in x , then $\Delta f(x)$ is a again a polynomial of degree $n - 1$ (5 marks)
- e) Set up Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (6 marks)
- f) Find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4} (6 marks)

QUESTION TWO (20 MARKS)

- a) Show that the operators Δ and E commute. (2 marks)
- b) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)
- c) Solve the equation $x^3 = \sin x$. Considering various $\phi(x)$, discuss the convergence of the solution. (5 marks)
- d) How accurately should the length and the time of vibration of a pendulum should be measured in order that the computed value of g be correct to 0.01% (8 marks)

QUESTION THREE (20 MARKS)

- a) Deduce the error in quotient and its absolute error (4 marks)
- b) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(5 marks)

- c) Find the cubic polynomial which takes the following values;
 $f(1)=24$, $f(3)=120$, $f(5)=336$, and $f(7)=720$. hence or otherwise, obtain the value of $f(8)$. (5 marks)
- d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Find the polynomial of lowest degree which assumes the values 10,4,40,424 and 620 at $x = -2, 1, 3, 7$ and 8. (6 marks)
- b) Construct the difference table for the following data and locate the error in the second column.

x	3.6	3.61	3.62	3.63	3.64	3.65	3.66	3.67	3.68
$f(x)$	0.1120 46	0.1202 04	0.1283 50	0.1364 62	0.1446 00	0.1527 02	0.1607 88	0.1688 57	0.1769 08

(7 marks)

- c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(7 marks)

QUESTION FIVE (20 MARKS)

- a) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation $f(x) = 0$. (10 marks)

- b) Use the method in a.) to find the root of the equation $\tan x = x$ near $x = 4.5$ correct to four decimal places. (10 marks)