MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ……………………

SMA 230: VECTOR ANALYSIS

DATE: **TIME:**

INSTRUCTIONS

Answer question one and Any other two questions

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define the following terms:

i. Mean Vector (1 mark)

ii. Random vector (1 mark)

b) List the three properties of Moment Generating Function (M.G.F). (3 marks)

c) Suppose that x_1, x_2, x_3 are random variables having a joint probability density function given by $f(x_1 x_2 x_3, x_4) = \begin{cases} 16x_1x_2x_3x_4, 0 \le x_i \le 1 \\ 0, elsewhere \end{cases}$. Determine the marginal distribution of:

i. $x_2 x_4$ (3 marks)

ii. x_2 (3 marks)

d) A three dimensional random vector x has a PDF given by

$f(x_1 x_2 x_3) = \begin{cases} 5x_1x_2x_3, 0 \le x_i \le 1, i = 1,2,3 \\ o, elsewhere \end{cases}$. Determine $E(2x_1 + 4x_2^2 + 3x_3^3)$ (4 marks)

e) Suppose $x \sim N(0,1)$. Find the characteristic function x. (4 marks)

f) Suppose $x_1, x_2 \cdots x_n$ are independent Bernoulli random variables with parameter, θ. Determine the distribution of $y = \sum_{j=1}^{n} x_j$. (4 marks)

g) Let x_1, x_2, x_3 be independent standard normal random variable. Using the change of variable technique determine the probability distribution $\underset{\sim}{y} = [y_1 y_2 y_3]'$ where $y_1 = x_1, y_2 = x_1 + x_2$ and $y_3 = x_1 + x_2 + x_3$. (4 marks)

h) Show that $\bar{X}_n \xrightarrow{p} \mu$ using Chebyshev's inequality. (3 marks)

SECTION B-ANSWER ANY TWO QUESTION IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Let x have the variance covariance matrix given by $\sum = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$. Find:

i. The correlation matrix of x (5 marks)

ii. The correlation coefficient between x_1 and $\frac{1}{3}x_2 + \frac{2}{3}x_3$ (4 marks)

b) Given the joint probability mass function of $x = [x_1, x_2, x_3]'$ as $f(x_1 x_2 x_3) = \begin{cases} k x_1 x_2 x_3, for 0 \le x_i \le 1, i = 1,2,3 \\ 0, otherwise \end{cases}$. Determine:

i. The value of k (5 marks)

ii. The marginal PDF of x_1 and x_2. (3 marks)

iii. The conditional distribution of $x_3 / x_1 = 1, x_2 = 2$ (3 marks)

QUESTION THREE (20 MARKS)

a) Briefly explain what is meant by quadratic form of $x_1, x_2 \cdots x_n$. Hence given that $x = [x_1, x_2, x_3]'$ has a mean vector and variance-covariance matrix given by $\mu = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and

$\sum = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ respectively. Find the expected value of the quadratic form

$Q = (x_2 - x_3)^2 + (x_1 - x_2 + x_3)^2$ (10 marks)

b) Suppose $x = [x, y]'$ has a two variate normal distribution with density $f(x) = c \exp -\frac{1}{2} Q$.

Where $Q = \frac{1}{7200}(1600x^2 + 900xy + 1125y^2 + 4140x + 450y + 5800)$. Identify $\mu, \sum$ and c.

(10 marks)

QUESTION FOUR (20 MARKS)

a) Let x_1, x_2, x_3 be independent and identically distributed random variables from a distribution whose probability density function is given by $f(x) = \begin{cases} e^{-x} for, x > 0 \\ o, elsewhere \end{cases}$. Using the change of variable technique determine the joint distribution of $Y_1 = \frac{x_1}{x_1 + x_2 + x_3}$ and $Y_2 = \frac{x_1 + x_2}{x_1 + x_2 + x_3}$.

Hence find the marginal distribution of $Y_2 / Y_1 = y_1$. (6 marks)

b) Given $M(t_1 t_2) = \left[(1 - p_1 - p_2) + p_1 e^{t_1} + p_2 e^{t_2}\right]^n$. Find:

i. $E(x_2)$ (2 marks)

ii. $E(x_1 x_2)$ (2 marks)

iii. δ_{12} (4 marks)

iv. ℓ_{12} (1 mark)

c) Consider a random vector $x = [x_1 x_2 x_3]'$ defined as $x \sim N(\underset{\sim}{\mu}, \sum)$ where $\underset{\sim}{\mu} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ and

$\sum = \begin{bmatrix} 11 & 5 & 2 \\ 5 & 4 & 1 \\ 2 & 1 & 3 \end{bmatrix}$. Find the mean and variance of the conditional density of x_1 given $x_2 = 5$ and $x_3 = 5$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let Z_n be a chi-square with n degrees of freedom. Find the limits distribution of $y = \frac{z_n - n}{\sqrt{2n}}$.

(10 marks)

b) Let $f(x)$ be a density function with mean μ and finite variance δ^2. Let $\overline{x_n}$ be the sample mean from $f(x)$ and let $\varepsilon > 0$ and $0 < \Delta < 1$ be any specified numbers.

i. If n is any integer greater than $\frac{\delta^2}{\varepsilon^2 \Delta}$ show that $p\left[-\varepsilon \leq \overline{x_n} - \mu \leq \varepsilon\right] > 1 - \Delta$.

(5 marks)

ii. How large a sample should be taken in order that you are 99% certain that $\overline{x_n}$ is within 0.5δ of μ. (3 marks)

c) A fair die is tossed 12 independent times. Determine the probability of the following combination: (2 marks)

Face	1	2	3	4	5	6
Frequency	2	3	0	2	4	1