MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 261: PROBABILITY AND STATISTICS II

DATE: 17/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE. COMPULSORY (30 MARKS)

a) Determine the $E(x_1 + x_2)$ from the Bivariate table below. (4 marks)

		x_2	
		0	1
x_1	0	$\frac{1}{6}$	0
	1	⅓	⅓
	2	0	$\frac{1}{6}$

b) If the joint probability function for y_1 and y_2 is given by $f(y_1, y_2) = \frac{1}{30}(y_1 + y_2)$

$y_1 = 0, 1, 2, 3.$

$y_2 = 0, 1, 2$

(i) Determine marginal distribution of y_1 and y_2 (7 marks)

(ii) Determine an expression $\emptyset(y_1 / y_2)$ the conditional distribution of y_1 given y_2 takes on all values of y_2. (3 marks)

c) The joint pdf of x and y is given by $f(x, y) = \boldsymbol{b}xy, for\ 0 < x, y < 1.$
Determine the value of ***b*** (5 marks)

d) Show that E(X+Y) =E(X) +E(Y) if X and Y are independent continuous random variables

(5 marks)

e) Given that $f(x, y) = \begin{cases} 2x, 0 < x, y < 1 \\ 0 \quad otherwise \end{cases}$

Determine $p(0 < x < ⅓, ½ < y < 1)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Let y be a continuous random variable with a pdf defined by that

$$f(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Using the change of variable technique determine the density of$x = 8y^3$. (8 marks)

b) If x and y have joint probability density function $f(x, y) = \frac{8x^2}{7y^3}$ for $1 \leq x, y \leq 2$

(i) Determine the marginal distribution of *x*. (4 marks)

(ii) Determine the conditional density function of *x* given $Y = y$ (6 marks)

(iii) Determine whether *x* and *y* are independent. (2 marks)

QUESTION THREE (20 MARKS)

a) Given that the joint density $f_{X,Y}$ is given by

$$f_{X,Y}(x, y) = \begin{cases} Cye^{-xy}, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

(i) Determine the value of C so that $f_{X,Y}$ is a joint density function. (4 marks)

(ii) Compute the marginal density of y (4 marks)

b) The joint pdf of x and y is given by that $f(x,y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) $\mu_x \; and \; \mu_y$ (6 marks)

(ii) $d_x^2 \; and \; d_y^2$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Let $y_1 \sim B(n_1, p)$ and $y_2 \sim B(n_2, p)$. Determine the moment generating function of $y_1 + y_2$ given that y_1 and y_2 are independent binomial random variables. (8 marks)

b) Suppose that x and y are discrete random variables with a p.d.f as shown in the bivariate table below:

		y		
		0	1	2
x	0	0.1	0.1	0.2
	1	0	0.15	0.05
	3	0.1	0.2	0.1

Determine the covariance of x and y i.e cov (x,y). (12 marks)

QUESTION FIVE (20 MARKS)

a) If the continuous random variable X with a pdf defined by that

$f_x(x) = \begin{cases} \frac{2}{9}(x+1), & -1 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$

Determine the p.d.f of $Y = X^2$. (10 marks)

(b) Given x and y have the joint probability density function (pdf) given by

$$f(x,y) = \begin{cases} K(x^2 + y^2), & 0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine (i) the value of the constant K. (3 marks)

(ii) the conditional p.d.f of x given for any value of y (5 marks)

(iii) $pr(x < \frac{1}{2} / y = \frac{1}{2})$ (2 marks)