



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN BIOLOGY

SMA 101: MATHEMATICS FOR SCIENCE II

DATE: 23/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 MARKS (COMPULSORY)

- a) Find the area bounded by the curve $y = 4 - x^2$ and the y-axis (5 marks)
- b) Evaluate the following limit
$$\lim_{x \rightarrow 1} (x^2 - 2x + 3)$$
 (2 marks)
- c) Evaluate the following integral
$$\int e^{x^5} x^4 dx$$
 (5 marks)
- d) Find the $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$ (3 marks)
- e) Find the limit of $\lim_{\theta \rightarrow \infty} \frac{\sin 2\theta}{\theta}$ (3 marks)
- f) Find the derivative of $f(x) = 2x^2 - x + 5$ from the first principal (3 marks)
- g) Find the derivative of $f(x) = \sin x$ using the 1st principal (5 marks)
- h) Integrate the following functions with respect to x
$$y = \tan^8 x \sec^2 x$$
 (4 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Obtain the derivative of $f(x) = 5x^2 - \cos x + 2$ (5 marks)
- b) Find $\frac{dy}{dx}$ for the following function
 $x = 3t^2 + 4t$ $y = e^{3t} + \cos^{-1} t$ (5 marks)
- c) Differentiate $4xy + (4x^4y^3 + 12xy^5 = 12 + 12y$ with respect to x (5 marks)
- d) Obtain the derivative of the following functions
 $f(x) = 3(2x + 2)^4 + x^3$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Calculate the maxima and minima values of function $y = x^3 - 3x^2 + 2$ and distinguish between them and sketch the graph. (5 marks)
- b) A cylinder is to be constructed so that the sum of height and ball radius is 6cm. Denoting ball radius by r cm, volume v cm³. Show that $r = \pi(6r^2 - r^3)$. Hence show the value of r which make V a maxima. (5 marks)
- c) Show that $\sin^2 \theta + \cos^2 \theta = 1$ (5 marks)
- d) Differentiate $\cos x$ using the first principle (5 marks)

QUESTION FOUR (20 MARKS)

- a) Find the integrals of the following function with respect to x , $y = \frac{2x+2}{x^2+2x+1}$ Evaluate the
 $\int_1^2 \ln x \, dx$ (5 marks)
- b) Evaluate the following integral using the given change of variable
 $\int \frac{x(x-4)}{(x-2)^2} \, dx$, $u = x - 2$ (5 marks)
- c) Evaluate the following integral
- i) $\int x^4 \sqrt{x^5 + 5} \, dx$ (5 marks)
- ii) $\int \sin 3x \cos 4x \, dx$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Find the area bounded by the $x = 9 - y^2$ and the y-axis (5 marks)
- b) Find the volume of the solid generated by the region $y = x^2 + 1$, x-axis and the lines $x=-1$ and $x=1$, rotated 360° about the x-axis. (5 marks)
- c) The region bounded by the y-axis, $y = x^3$ $y = 1$ and $y = 8$ is rotated 360° about the y-axis. Find the volume of the resulting solid. (5 marks)
- d) Find the volume generated by revolving the region bounded by $y = \sqrt{x}$, the lines $x = 1$ and $x = 4$ about the line $y = 1$ (5 marks)