MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SSEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 203: LINEAR ALGEBRA II

DATE: 12/4/2023 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer QUESTION ONE and any OTHER TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Show that the zero function $T: V \to W, T(x) = 0 \ \forall x \in V$ is linear (4 marks)

b) Determine whether or not $T_1(x, y, z) = (2x + y, xy, x - y)$ is a linear transformation (4 marks)

c) Show that :$T(x, y) = x^2 + y^2$ is not linear (4 marks)

d) Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x) = T(x_1, x_2) = (x_1, -x_2)$ is one-to-one and onto. (4 marks)

e) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x + y, x + y)$. Find the rank, nullity, Kernel and basis of $Ker\ T$ and $Im\ T$. (5 marks)

f) Determine the matrix representation of $T: \mathbb{R}^2 \to \mathbb{R}^2$defined by $T(x, y) = (2y, 2x - y)$ relative to the standard basis $e_1 = (1,0), e_2 = (0,1)$ of $\mathbb{R}^2$ (5 marks)

g) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x + y, -2x + 4y)$. Find the determinant of T. (4 marks)

QUESTION TWO (20 MARKS)

a) Let $T(x, y) = (x + 4y, -2x + 3y)$and let $\{e_1 = (1,0), e_2 = (0,1)$and $f_1 = (1,1), f_2 = (2, -1)$ } be two basis of $\Re^2$

i) Determine the transition matrix P from e_i to f_i and the transition matrix $Q = P^{-1}$, from f_i to e_i

ii) Show that $P^{-1}[T]_e P = [T]_f$, is a diagonal matrix of T i.e T is diagonalizable.

(10 marks)

b) Let $= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $f(t) = 2t^2 - 3t + 7$ and $g(t) = t^2 - 5t - 2$. Find $f(A) and\ g(A)$.

(10 marks)

QUESTION THREE (20 MARKS)

a) Determine the eigen values and eigen vectors associated with the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

. (7 marks)

b) Find all eigen values and a basis for each eigen space of T: $\Re^2 \to \Re^2$ defined by

$T(x, y, z) = (2x + y, y - z, 2y + 4z)$ (6 marks)

c) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Find all the eigen values of A and a basis for each eigen space.(7 marks)

QUESTION FOUR (20 MARKS)

a) Diagonalize $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ (6 marks)

b) For each matrix find all the eigen values and a basis of each eigen space

$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} -3 & 1 & -1 \\ 7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$

Which matrix can be diagonalized and why? (14 marks)

QUESTION FIVE (20 MARKS)

a) Let T: $\mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ be defined by $T(x, y) = (2x + y, -2x + 4y)$. Find the determinant of T. (5 marks)

b) Let $T: R^2 \rightarrow R$ be defined by $T(x, y) = x + y + 1$ .Show that T is not linear. (5 marks)

c) Show that the mapping defined by $T(x, y, z) = (2x + y - 4z)$ where $T: R^2 \rightarrow R$ is a linear map. (5 marks)

d) Show that $T: R^2 \rightarrow R$, defined by $T(x, y) = x^2 + y^2$, is not one-one. (5 marks)