MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
BACHELOR OF SCIENCE IN MATHEMATICS

SMA 461: OPERATIONS RESEARCH II

DATE: 21ST JANUARY, 2025 **TIME: 2.00 PM – 4.00 PM**

INSTRUCTIONS:

1. *Answer* ***Question* 1** *and any other* ***two*** *questions.*
2. *You must have a scientific calculator and statistical tables for this paper:*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) Outline *three* examples from real-life situations of areas of application of each of the following as a problem-solving technique in operational research:

(i) queuing theory;

(ii) network flows. (6 *marks*)

(b) (i) Explain the term *crashing* as used in project network analysis, giving at one example. (3 *marks*)

(ii) Differentiate between the terms CPA and PERT as used in project network analysis. (3 *marks*)

(iii) Explain two *constraints* which are applicable in the optimisation of a network flow using the maximum flow minimum cut technique. (4 *marks*)

(c) A certain university grades its students using letters A, B, C and D. From past records, it is known that the academic performance as observed in the grades scored by the students has the following probabilities: Grade A is 0.15, Grade B is 0.35, Grade C is 0.30, and Grade D is 0.20. From a random sample of 45 students, the university intends to estimate the expected number of students who will score each grade. The performance of students is random and independent of each other.

Solve the problem using Monte Carlo simulation technique. (6 *marks*)

Use the random numbers below representing the marks scored:

19	39	58	56	06	97	43	71	70	38	99	36	81	36	46
63	30	62	05	04	43	96	06	55	56	30	48	76	27	50
60	84	70	33	13	32	03	09	94	54	73	96	84	43	49

(d) Below is the precedence table for the project of building a boat.

Activity	Preceding Activity	Activity Description
A	–	Design hull
B	–	Prepare boat shed
C	A	Design mast and mast mount
D	A	Obtain hull
E	A	Design sails
F	C	Obtain mast mount
G	C	Obtain mast
H	C	Design rigging
J	B, D	Prepare hull
K	F, J	Fit mast mount to hull
L	E, H, G, K	Step mast
M	E, H	Obtain sails and rigging
N	L, M	Fit sails and rigging

Construct a network to represent this data. (8 *marks*)

QUESTION TWO (20 MARKS)

(a) In a certain donor funded institution, a bank cheque must be signed by three officers: Chief Executive Officer, Chairman and Finance officer. The probabilities of the three signatories being available during any given week are as follows: Chief Executive Officer 0.85, Chairman 0.70, and Finance Officer 0.95. Cheques are drawn at random as need arises, and the three signatories are available at random and independent of each other.

Simulate this problem using Monte Carlo method. Hence, taking a period of 15 weeks, determine the expected (mean) number of weeks for which a cheque will be fully signed by all the signatories who are available.

Use the set of random numbers below: (10 *marks*)

CEO	60	47	87	86	00	47	27	36	12	99	61	30	76	87	40
Chairman	45	57	93	74	59	52	58	98	20	65	46	94	25	81	67
Finance Officer	38	60	19	65	90	70	21	25	38	21	33	34	59	82	85

(b) The data given below is a utility matrix representing 5 job applicants and 5 job positions in a certain organisation.

$$\mathbf{A} = \begin{bmatrix} 7 & 2 & 8 & 4 & 3 \\ 5 & 4 & 6 & 3 & 3 \\ 6 & 3 & 7 & 4 & 5 \\ 5 & 2 & 5 & 2 & 4 \\ 3 & 0 & 4 & 3 & 1 \end{bmatrix}$$

Determine the optimal assignment of the applicants to the positions in such a way that will maximise the total utility. (10 *marks*)

QUESTION THREE (20 MARKS)

Machakos University in collaboration with an organisation has developed a plan to set up an ICT laboratory. The laboratory project has been broken down into identifiable activities, with their preceding activities and activity duration using multiple time estimates as shown in the precedence table below:

Activity	Preceding Activity	Duration in Days		
		Optimistic	Most Likely	Pessimistic
A	–	06	06	24
B	–	06	12	18
C	A	10	16	34
D	A	07	07	13
E	B	06	12	18
F	B	08	14	20
G	D, E	06	12	18
H	D, E	12	12	30
J	F	06	09	24
K	C, G	08	14	20
L	C, G	06	09	24
M	H, K	06	12	18
N	J	06	06	24
P	L, M	08	14	20
Q	N, P	06	12	18

(i) Construct a network diagram to represent the data in the precedence table. (6 *marks*)

(ii) For each activity, determine the expected duration and standard deviation of the duration. (3 *marks*)

(iii) Determine the critical path of the network and the expected project duration. (4 *marks*)

(iv) Determine the probability of completing the project in 93 days. (3 *marks*)

(v) Determine the *total float* for the two longest non-critical chain of activities in the network. (4 *marks*)

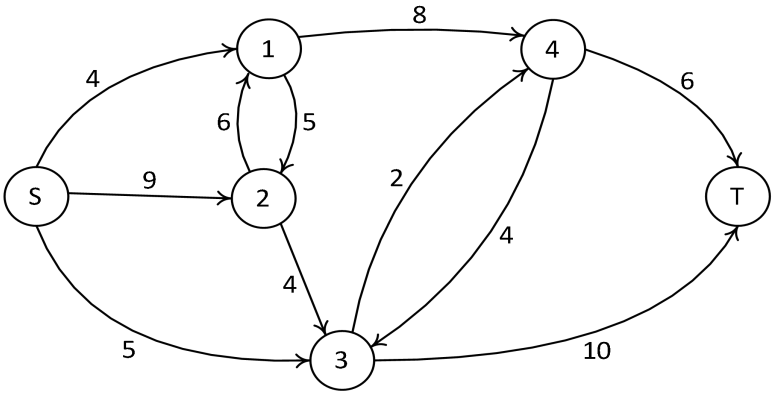
QUESTION FOUR (20 MARKS)

(a) (i) Outline *four* real-life examples of network flows as used in operations research.

(2 *marks*)

(ii) Outline two examples of areas of application of the *assignment theory* as an operational research technique. (2 *marks*)

(b) Given below is a network diagram representing network flows.

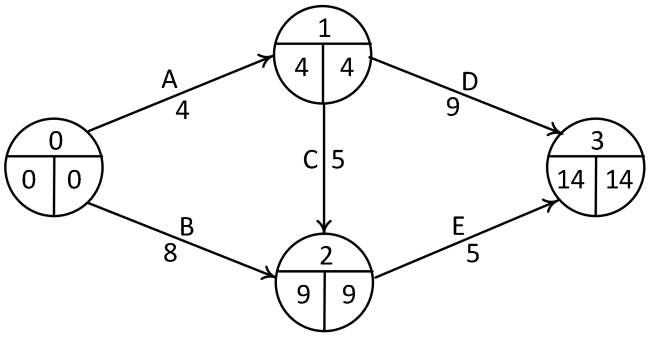


Determine the maximum flow and the minimum cut in the flow network. (16 *marks*)

QUESTION FIVE (20 MARKS)

(a) A project comprising 5 activities is represented with the following data comprising time in weeks and cost in thousand Kenya shillings for both the normal and crash situations. Using the corresponding network, prepare the least cost schedules for all possible durations from normal time and normal cost to crash time and crash cost.

Activity	Preceding activity	Time		Cost		Cost slope
		Normal	Crash	Normal	Crash	
A	–	4	3	360	420	60
B	–	8	5	300	510	70
C	A	5	3	170	270	50
D	A	9	7	220	300	40
E	B, C	5	3	200	360	80



(12 *marks*)

(b) A company has 5 workers W_1 , W_2 , W_3 , W_4 and W_5 and 5 tasks T_1, T_2, T_3, T_4 and T_5. From past experience, the ability of the workers to perform the tasks are as follows:

Worker	**Tasks**
W_1	T_2
W_2	T_1 , T_4
W_3	T_1 , T_2
W_4	T_1 , T_2 , T_3 , T_4
W_5	T_3 , T_5

(i) Represent the above information in a bipartite graph and a binary matrix.

(ii) Determine the maximum match for the assignments. (8 *marks*)