MACHAKOS UNIVERSITY

University Examinations 2019/2020Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 402: QUANTUM MECHANICS II

DATE: 10/12/2019 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS)

a) (i) State three basic operators generally used in quantum mechanics. (3 marks)

(ii) Define $J^2 = J_x^2 + J_y^2 + J_Z^2$ (The symbols carry their usual meaning). (1 mark)

b) Define the term coupled particles with reference to the dynamics of the system (1 mark)

c) Distinguish between antisymmetric and symmetric wave function (use an expression) (4 marks)

d) Define the term 'selection rules' as a concept used in quantum mechanics (1 mark)

e) i) Define the term orbital angular momentum **L** as applied in quantum mechanics (1 mark)

ii) Show that $[J^2, J_K] = 0$ (3 marks)

f) i. Show that the wave function ψ vanishes (equal to zero) if the two particles occupy the same location for antisymmetric wave function (3 marks)

ii. Show that there's a probability of finding both the particles described by the symmetric wave function of the same location (2 marks)

g) Define the term Spin as used in quantum mechanics (1 mark)

h) i. State Pauli Exclusion principle (1 mark)

ii. Distinguish between Fermi-Dirac statistics and Bose-Eistein statistics giving an example of particle belonging to each (4 marks)

i) Obtain the expectation values: (5 marks)

$$\bar{S}_Z = \langle 0 | \hat{S}_z | 0 \rangle = \frac{1}{2}\hbar,$$

$$\bar{S}_Z = \langle 1 | \hat{S}_z | 1 \rangle = -\frac{1}{2}\hbar$$

where S_z is the z component of the spin angular momentum eigenvalues $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$

SECTION B

QUESTION TWO (20 MARKS)

a) Define a spin-½ system (1 mark)

b) Distinguish between the spin upstate and spin downstate (3 marks)

c) Obtain the commutation brackets: (9 marks)

(i) $[\sigma_+, \sigma_z]$

(ii) $[\sigma_-, \sigma_z]$

(iii) $[\sigma_+, \sigma_-]$

d) Show that the commutation relations for the angular momentum operators is expressed in the final form:

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z.,$$

$$[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x.,$$

$$[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y.$$

where orbital angular momentum can be expressed in matrix form as

$$\hat{L} = (\hat{y}\hat{p}_z - \hat{z}\hat{p}_y)\mathrm{i} + (\hat{z}\,\hat{p}_x - \hat{x}\,\hat{p}_z)\hat{\mathrm{j}} + (\hat{x}\,\hat{p}_y - \hat{y}\,\hat{p}_x)\hat{\mathrm{k}}.$$

(7 marks)

QUESTION THREE (20 MARKS)

To capture the spherical symmetry of the three-dimensional dynamics, the orbital angular momentum operators L_x, L_y, L_z can be expressed in spherical polar coordinates (r, θ, ϕ).

a) State the physical meaning of the orbital angular momentum operators L_x, L_y, L_z (3 marks)

b) Express orbital angular momentum operators in spherical polar coordinates (9 marks)

c) Use the results to determine $L^2 = L_x^2 + L_y^2 + L_Z^2$ in spherical polar coordinates (8 marks)

QUESTION FOUR (20 MARKS)

a) Find the degeneracy of energy state n=3. (9 marks)

b) Work out clearly the possible combination of *n*, *l* and *ml*, that will give rise to the generacy of this level (9 marks)

c) Explain why the level n=3 is said to be a 9 fold degenerate (2 marks)

QUESTION FIVE (20 MARKS)

a) Obtain the commutation brackets: (10 marks)

$[\mathrm{J}_x, \mathrm{J}_y] = iJ_z.,$

$[\mathrm{J}_y, \mathrm{J}_z] = iJ_x.,$

$[\mathrm{J}_z, \mathrm{J}_x] = iJ_y.$

b) Given $|ab\rangle$ to be an eigenket of belonging to an eigenvalue *b-h*,

show that $\hat{J}_z\left(\hat{J}_-|ab\rangle\right) = (b-\hbar)\left(\hat{J}_-|ab\rangle\right)$ (10 marks)