MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONIC ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 300: ENGINEERING MATHEMATICS IX

DATE: 10/8/2021 **TIME: 8:30 -10:30 AM**

INSTRUCTIONS

Answer question **one** (**Compulsory**) and any other **two** questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find unit vectors normal to the surfaces $x^2 + y^2 - z^2 + 3 = 0$ and $xy - yz + zx - 10 = 0$ at the point (3, 2, 4) and hence find the angle between the two surfaces at that point. (4 marks)

b) Prove that $\nabla^2 \left(\frac{1}{r}\right) = 0$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ (5 marks)

c) Prove the vector triple product $\boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) = (\boldsymbol{b}.\boldsymbol{a})\boldsymbol{c} - (\boldsymbol{b}.\boldsymbol{c})\boldsymbol{a}$ (4 marks)

d) Determine the unit base vectors in the directions of the following vectors and determine whether the vectors are orthogonal:

$2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}, 4\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ (4 marks)

e) Find the directional derivative of $\varphi = (x + 2y + z)^2 - (x - y - z)^2$ at the point (2,1,-1) in the direction of $\boldsymbol{A} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$. (4 marks)

f) Using Divergence theorem, evaluate $\iint_S \vec{A}\,.\vec{n}ds where \vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, z=0 to z=3. (4 marks)

g) Given the space curve $x = 2t, y = t^2 \ and \ z = \frac{2}{3}t^3$ find;

(i) Curvature k

(ii) Torsion τ. (5 marks)

QUESTION TWO (20 MARKS)

a) Given that $\vec{R} = e^{-t}\boldsymbol{i} + \ln(t^2+1)\,\boldsymbol{j} + \tan t\,\boldsymbol{k}$, determine;

i) $\left|\frac{d\vec{R}}{dt}\right|$ at $t = o$ (3 marks)

ii) $\left|\frac{d^2\vec{R}}{dt^2}\right|$ at $t = 0$ (3 marks)

b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t \ and \ z = 3t - 5$, determine the components of its velocity and acceleration at a time $t = 1$ in the direction of $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ (7 marks)

c) Given that $\vec{A} = x^2yz\boldsymbol{i} - 2xz^3\boldsymbol{j} + xz^2\boldsymbol{k}$and $\vec{B} = 2z\boldsymbol{i} + y\boldsymbol{j} - x^2\boldsymbol{k}$, determine $\frac{\partial^2}{\partial x \partial y}\left(\vec{A} \times \vec{B}\right)$ at $(1,0,-2)$ (7 marks)

QUESTION THREE (20 MARKS)

a) If V is a scalar field such that $V = u^2vw^3$ and scale factors are $h_u = 1, h_v = 2 \ and \ h_w = 4$, determine $\nabla^2 V$ at the point (2, 3,-1). (5 marks)

b) Prove the following Frenet-Serret formulae:

$$\frac{d\vec{B}}{dS} = -T\vec{N}$$

(7 marks)

c) If $\phi = 2xyz^2$ and **F**=xy**i**-z**j**+x^2**k** and C is the curve $x=t^2$, $y=2t$ and $z=t^3$ from t=0 to t=1; evaluate the line integrals:

i) $\int \phi d\underset{\sim}{r}$

ii) $\int_c \underset{\sim}{F} \times d\underset{\sim}{r}$ (8 marks)

QUESTION FOUR (20 MARKS)

a) Find the equation of the tangent plane to the plane surface $2xz^2 - 3xy - 4x = 7$ at the point $(2, -2, 3)$ (4 marks)

b) Find the directional derivative $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ (4 marks)

c) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2\, dy$ where c is the closed curve of the region bounded by y=x and $y=x^2$. (6 marks)

d) Given that $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{w} = w_1\boldsymbol{i} + w_2\boldsymbol{j} + w_3\boldsymbol{k}$ and $\vec{r} = \boldsymbol{xi} + y\boldsymbol{j} + z\boldsymbol{k}$, prove that $\vec{w} = \frac{1}{2} curl\, \vec{v}$. (6 marks)

QUESTION FIVE (20 MARKS)

a) Find the work done in moving a particle in a force field given by $\boldsymbol{F} = 3xy\boldsymbol{i} - 5z\boldsymbol{j} + 10x\boldsymbol{k}$ along the curve $x = t^2 + 1; y = 2t^2; z = t^3$ from t=1 to t=2. (6 marks)

b) If **F**=2xz**i**-x**j**+y^2**k** , evaluate $\iiint_V \vec{F}\, dv$ where V is the region bounded by the surfaces x=0, $x = 2$ y=0, y=6, $z=x^2$ and z=4. (7 marks)

c) Evaluate $\iint_S \vec{A} . \vec{n} ds$ where A=zi+xj-3zy^2 and s is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z=0 and z=5. (7 marks)