MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SPECIAL EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA231 DYNAMICS I

DATE: 13/4/2023 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question one (**compulsory**) and any other **two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) A particle of mass 2 units moves along a space curve defined by $\boldsymbol{r} = (4t^2 - t^3)\boldsymbol{i} - 5t\boldsymbol{j} + (t^4 - 2)\boldsymbol{k}$. At a time $t = 1$, find;

(i) Momentum, (2 marks)

(ii) Force, (2 marks)

(iii) Power. (2 marks)

b) Under the influence of a force $\boldsymbol{F}$, a particle of mass m moves along the ellipse defined by $\boldsymbol{r} = a\cos\omega t\,\boldsymbol{i} + b\sin\omega t\,\boldsymbol{j}$, given that $\boldsymbol{r}$ is the position vector of the particle and $\boldsymbol{p}$ is its momentum at any time t, prove that;

(i) $\boldsymbol{r}\cdot\boldsymbol{p} = \frac{1}{2}m\omega(b^2 - a^2)\sin 2wt$ (3 marks)

(ii) $\boldsymbol{r}\times\mathbf{p} = mab\omega\boldsymbol{k}$ (4 marks)

c) Due to a force field $\boldsymbol{F}$, a particle of mass 4 units moves along a space curve $\boldsymbol{r} = (3t^2 - 2t)\boldsymbol{i} + t^3\boldsymbol{j} - t^4\boldsymbol{k}$. Find the work done by the field moving the particle from a point when $t = 1$ to another point when $t = 2$. (3 marks)

d) A body of a unit mass moves in a force field $\boldsymbol{F} = (3t^2 - 4t)\boldsymbol{i} + (12t - 6)\boldsymbol{j} + (6t - 12t^2)\boldsymbol{k}$. At a time $t = 2$, find,

(i) Torque and (2 marks)

(ii) Angular momentum. (2 marks)

e) A body of mass m is vertically projected upwards from the earth's surface with a speed v_0. Derive an expression for

(i) Position of the mass at any time, (2 marks)

(ii) Time taken to reach maximum height, (1 marks)

(iii) The maximum height attained. (2 marks)

f) A simple pendulum bob of mass m attached to a massless inextensible string of length l is displaced through a small angle θ and allowed to oscillate freely.

(i) Write the equation of motion for the motion assuming there is no damping and no external forces are present. (1 mark)

(ii) Show that the solution of the equation in (i) above is given by

$$\theta = \theta_0 \cos \sqrt{\frac{g}{l}}\, t$$ (2 marks)

g) A mass m_1 travelling with a speed v on a horizontal plane hits another mass m_2 which is at rest. If the coefficient of restitution is ε, prove that there is loss of kinetic energy equal to $\frac{m_1 m_2 (1-\varepsilon^2) v^2}{m_1 + m_2}$ (2 marks)

QUESTION TWO (20 MARKS)

a) The force $\boldsymbol{F}$ acting on a particle of mass m expressed in terms of time t is given by $\boldsymbol{F} = a\cos\omega t\,\boldsymbol{i} + b\sin\omega t\,\boldsymbol{j}$, if the particle is initially at rest at the origin, find an expression for;

(i) Its position at any time, (6 marks)

(ii) Velocity at any time. (4 marks)

b) A particle is of mass m, is travelling along the x-axis such that at a time $t = 0$, it is located at $x = 0$ and has a speed v_0. The particle is acted upon by a force $\boldsymbol{F}$ which opposes motion and and whose magnitude is proportional to the square of the instantaneous speed v given by $\boldsymbol{F} = -\beta v^2 \boldsymbol{i}$. Find the expression for;

(i) Speed, (4 marks)

(ii) Position, (3 marks)

(iii) Acceleration, of the particle at any time $t > 0$. (3 marks)

QUESTION THREE (20 MARKS)

a) A particle is moving in a force field,

$$\boldsymbol{F} = (y^2 - 2xyz^3)\boldsymbol{i} + (3 + 2xy - x^2z^3)\boldsymbol{j} + (6z^3 - 3x^2yz^2)\boldsymbol{k}$$

(i) Prove that the force field is conservative (4 marks)

(ii) Find the potential V associated with the force (3 marks)

(iii) Find the work done in moving the particle from $(1, 0, 0)$ to $(4, 6, 8)$ (3 marks)

b) A projectile is launched with an initial velocity v_0 at an angle α to the horizontal. Find an expression for;

(i) Position vector at any time, (2 marks)

(ii) Time of the flight, (2 marks)

(iii) Range, (2 marks)

(iv) Show that the path of the projectile is a parabola, (2 marks)

(v) Show that the range of the projectile is maximum when launched at 45^0 (2 marks)

QUESTION FOUR (20 MARKS)

a) A body of mass m slides without rolling down in an inclined plane of length l and having a constant coefficient of friction μ inclined at an angle α to the horizontal. If it starts from rest at the top of the inclined plane, find an expression for;

(i) Time τ taken for the body to reach the bottom of the plane. (6 marks)

(ii) The speed of the body at the bottom of the plane. (4 marks)

b) A spring of negligible mass suspended vertically from one end is stretched a length of 20cm when a mass of 5g is attached to the other end. The spring and the mass are placed horizontally on a frictionless table. The mass is pulled down a length of 20cm and released to oscillate freely. Find;

(i) The differential equation describing the motion and its equivalent initial conditions. (2 marks)

(ii) Position of the mass at any given time. (4 marks)

(iii) Amplitude, period and frequency of the oscillations. (4 marks)

(take g = 9.8N/kg)

QUESTION FIVE (20 MARKS)

a) Two masses m_1 and m_2 travelling on a straight line with velocities v_1 and v_2 respectively collide. Using Newton's collision rule, $v' = \varepsilon v$, obtain expressions of the velocities of the masses after collision $v_1{}'$ and $v_2{}'$ in terms of the velocities before collision, for;

(i) Perfectly inelastic collision (6 marks)

(ii) Perfectly elastic collision. (4 marks)

b) Considering a simple harmonic oscillator,

(i) Show that the wave motion is defined by the equation $x(t) = C\cos(\omega t - \phi)$ where C is the amplitude and ϕ is the phase angle. (6 marks)

(ii) Is $\phi = 0$, sketch the wave motion and define the period. (4 marks)