MACHAKOS UNIVERSITY

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UNIVERSITY EXAMINATIONS 2019/2020

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICCS, MATHEMATICS AND COMPUTER SCIENCE, EDUCATION SCIENCE AND ARTS

SMA 406: FUNCTIONAL ANALYSIS

NSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

Question one 20 marks

a) Define a vector over a field K (5 marks)

b) Prove that in the set $\mathbb{R}$ of all real numbers define d on $\mathbb{R}x\mathbb{R}$ by

$d(x, y) = |x - y|\ \forall\, x, y \in \mathbb{R}$, d is a metric space. (5 marks)

c) Take $X = (0, 1]$ as a sequence of $\mathbb{R}$. Show that X is not complete. (5 marks)

d) Show that $||x|| = (\sum_1^\infty |x_n|^p)^{\frac{1}{p}} \geq 0$ is a norm in l_p. (5 marks)

e) Show that $Tx = 2x - 1$ is not a linear operator. (5 marks)

f) Find the fixed points of the mapping $T: \mathbb{R} \to \mathbb{R}^2$ defined by $Tx = x^2$ (5 marks)

Question two 20 marks

a) Prove that $Tx = \frac{1}{x}$ is not uniformly continuous on the interval (0,1). (5 marks)

b) Show that $p(x) = |x|$ is a semi-norm on $\mathbb{C}$. (5 marks)

c) Prove that if $T: X \to Y$, defined by $T(x) = 3x$, then T is linear. (5 marks)

d) Find the fixed points of the mapping $T: \mathbb{R} \to \mathbb{R}$ defined by $Tx = 3x^2 - 4$. (5 marks)

Question three 20 marks

a) Let T be a bounded linear operator, then

i) Show that if $x_n \to x$ [where $x_n, x \in D(T)$] then it implies that $T: x_n \to Tx$,

(5 marks)

ii) The kernel of T is closed. (5 marks)

b) Prove that if Y is a banach space then $B(X, Y)$ is a banach space. (5 marks)

c) Prove that a compact subset M of a metric space is closed and bounded. (5 marks)

Question four 20 marks

a) Show that $\int_a^b x(t)dt$, $x \in c[a, b]$ is a linear and bounded function on $c[a, b]$. (5 marks)

b) Prove that if $T: X \to X$, where X is a normed linear space is continuous at the origin, then T is uniformly continuous. (5 marks)

c) Let E be an inner product space. Then show that for arbitrary $x, y \in E$

$$\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$$ (5 marks)

d) Prove that if X and Y are Banach spaces and $T: T: X \to Y, D(T) \to Y$, is a closed linear operator, where $D(T) \subset X$. Then $D(T)$ is closed in X, the operator T is bounded.

(5 marks)