MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 303 – RING THEORY

DATE: 22/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One And Any Other Two Questions

SECTION A: (COMPULSORY)
QUESTION ONE(30 MARKS)

a) Define a ring as used in mathematics (4 marks)

b) Find the addition and multiplication tables for Z_6 (4 marks)

c) Show that in a commutative ring R with unity $1 \neq 0$ an ideal P is prime if and only if the factor ring R/P is an integral domain (6 marks)

d) Let $\emptyset : R \rightarrow R$ be a ring map proof that $\emptyset$ injective IF and only IF $\ker\emptyset = \{0\}$proof (4 marks)

e) Show that if I is an ideal of a ring R, then the canonical map T: R $\rightarrow$ R/I given by T(a) = a + I for a $\in$ R is a ring homomorphism (4 marks)

f) Proof the theorem which states that a multiplicative inverse of a unit x of a ring with unity 1 is unique (4 marks)

g) Which of the following rings is a field?

(i) $\left[a + b\sqrt{2} \backslash a, b \in Q\right] = A$ (2 marks)

(ii) $\left[a + b\sqrt{2} \backslash a, b \in z\right] = B$ (2 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIONS

QUESTION TWO (20 MARKS)

Given that S = $\{(a, b) \backslash a, b \in Q\}$ with operations addition (+) and multiplication (.) forms a ring where the operations + and defined by

(a, b) + (c, d) = (a + c, b + d)

(a, b). (c,d) = (ac + bc + ad, bd).

i) Show that S is commutative with unity e (5 marks)

ii) Find all the elements in S of the form x.x = e (6 marks)

iii) Determine the multiplicative inverse for (a, b) in S where it exist and a relation between a and b when the inverse does not exist (6 marks)

iv) Is S a field. Give reasons (3 marks)

QUESTION THREE (20 MARKS)

a) Let f: C → M_2(IR) be defined by f(a + bi) = $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ show that f is a ring homomorphism (6 marks)

b) Show that all ideals in the ring Z of integers are principal ideals (8 marks)

c) The polynomial $x^4 + 3x^3 + 2x + 4 \in Z_5(x)$ can be factorised into linear factors. Find this factorization (6 marks)

QUESTION FOUR (20 MARKS)

a) Show that a field is an integral domain (6 marks)

b) i) Define a nilpotent element in a ring R (2 marks)

ii) Find all the nilpotent elements in

Z_{12} (2 marks)

Z_{24} (3 marks)

c) Proof that all ideals in the ring Z of integers are principal ideals that is Z is a principal ideal domain (7 marks)

d) Given that r is a commutative ring with unity. Then M is a maximal ideal of R if and only if R/M is a field (7 marks)

QUESTION FIVE (20 MARKS)

a) i) Define the meaning of irreducible polynomials (3 marks)

ii) Show that $f(x) = x^3 + 3x + 2$ in $Z_5[x]$ is irreducible are Z_5 (6 marks)

b) Let $f(x) \in F[x]$ and let f(x) be of degree 2 or 3. Proof that f(x) is reducible over F if and only it has a zero in F (7 marks)

c) Suppose $f:R_1 \rightarrow R_2$ is a ring homomorphism if R_1 possesses unity 1 and R_2 possesses unity 1^1 then $f(1) = 1^1$ (4 marks)