MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SPECIAL EXAMINATION FOR BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 361: THEORY OF ESTIMATION

DATE: **TIME:**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)

(a) (i) A random sample of five values are taken from a normal population with mean μ and variance δ^2both unknown. The values are: 1.07, 1.02, 1.04, 1.06, and 1.05. Estimate the population mean and population variance. (3 marks)

(ii) $Let\ x_1, x_2, .. x_n$be a random sample from a population with mean μ and variance δ^2. Consider the following estimators for μ.

$$t_1 = {}^1\!/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
- Determine the efficiency of t_3 relative to t_2 (5 marks)

(b) (i) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, x=0,1,….

Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ (4marks)

(ii) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ. (5 marks)

(c) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{cases} (\lambda+1)x^{\lambda}, & 0 < \mathrm{x} < 1,\ \lambda > 0 \\ 0 & elsewhere \end{cases}$$

Obtain the moment estimator of λ (8 marks)

QUESTION TWO (20 MARKS)

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (8 marks)

(b) $Let\ x_1, x_2, .. x_n$be a random sample of size n from a population whose pdf is given by

$f(x, \theta) = \theta e^{-\theta x}$,x> 0,

Determine the Cramer-Rao lower bound for θ. (12 marks)

QUESTION THREE (20 MARKS)

Three objects M, N and P with weights$t_{1,}t_2\ and,t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$ (20 marks

QUESTION FOUR (20 MARKS)

(a) A random sample of size n is drawn from a normal population with (0,δ^2). Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (10 marks)

(b) Let $x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf if $f(x, \theta)$where θ is unknown. Let $T = t(x_1, x_2, ... x_n)$ be the necessary and sufficient condition for T to be the Minimum variance unbiased estimator (MVUE) of θ expressed in the form

$$\frac{\partial LogL}{\partial \theta} = \frac{T - \theta}{\lambda}$$

Show that T is MVUE of θ and $Var(T) = \lambda$ (10 marks)

QUESTION FIVE (20 MARKS)

(a) Let $x_1, x_2, .. x_n$ be a random sample from $x \sim N(\mu, \delta^2)$, obtain $100(1-\propto)\%$ confidence interval for μ, when δ^2 is known . (8 marks)

(b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased. (12 marks)