



MACHAKOS UNIVERSITY

University Examinations for 2018/2019 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

FIRST YEAR SEMESTER EXAMINATION

FOR BACHELOR OF ARTS

EES 100: MATHEMATICS FOR ECONOMISTS I

DATE:

TIME:

INSTRUCTIONS: Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Consider the following consumption and tax functions:

$$C = 70 + 0.85 Y^d$$

$$T = \alpha_0 + \alpha_1 Y$$

$$Y^d = Y - T$$

i) Identify the endogenous and exogenous variables (2 marks)

ii) If income increases by 100, how much of that increase will be allocated to consumption (2 marks)

b) Consider the following universal set T and its subsets C, D and E:

$$T = \{0, 2, 4, 6, 8, 10, 12\}$$

$$C = \{4, 8\}$$

$$D = \{10, 2, 0\}$$

$$E = \{0\}$$

Find $C \cap D \cap E$ (4 marks)

c) Consider the multivariate demand functions

$$Q_m = 2.2 - 1.8P_m + 1.2P_n - 0.7Y$$

$$Q_n = 1.7 - 1.1 P_m - 1.3 P_n - 0.9Y$$

- i) Are good “m” and “n” complementary or substitutes? (1 mark)
- ii) Is good “m” normal or inferior? Explain (1 mark)
- iii) Is good “n” normal or inferior? Explain (1 mark)

d) A single commodity market model is defined by the following:

$$Q_d = 16 - 4P$$

$$Q_s = -10 + 6P$$

$$\bar{Q}_d = \bar{Q}_s = \bar{Q}$$

where Q_d is the quantity of the commodity demanded Q_s is the quantity of the commodity supplied and P is the price of the commodity:

- i) At what price is equilibrium attained in the market? (2.5 marks)
- ii) Find the equilibrium quantity supplied (2.5 marks)

e) Given the following average total costs and the average revenue functions

$$3 + ATC - \frac{15}{Q} - Q = ATC$$

$$AR - \frac{4}{9}Q = 12$$

Determine the total costs, average variable costs, average fixed costs and total revenue functions (4 marks)

f) The commodity and money markets for an economy are defined by the following equation:

$$Y = c + I$$

$$C = 200 + \frac{2}{5}Y$$

$$I = 1900 - 12r$$

Money market

$$MDT = \frac{1}{2}Y$$

$$MD_s = 100 - 10r$$

$$M_s = 1500$$

- i) Derive the IS and the LM functions of the economy. (4 marks)
- ii) What is the equilibrium income and the rate of interest for the economy (4 marks)

g) The marginal propensity to consume is:

$MPC = 0.8 + \frac{1}{8}Y^{-\frac{1}{3}}$. Determine the corresponding consumption and saving functions? (2 marks)

QUESTION TWO (20 MARKS)

Write short notes on the following topics as used in mathematics for economists. Illustrate your work where necessary.

- i) The laws of set Algebra
- ii) Autonomous tax and Induced tax
- iii) Production transformation curve
- iv) Normal and Inferior goods
- v) IS-LM models
- vi) Relationship between price elasticity of demand and marginal revenue
- vii) Minimization of average variable cost
- viii) Demand and supply of money
- ix) Partial and General equilibrium
- x) Import and export functions.

QUESTION THREE (20 MARKS)

- a) Prove that $P = \frac{1}{\alpha Q}$ demand function has a unitary elasticity (3 marks)
- b) The AR and TC for a firm are given by:

$$AR = 5\frac{1}{2} - \frac{1}{2}Q$$

$$TC = \frac{1}{20}Q^3 - \frac{3}{10}Q^2 + 2Q$$

Find:

- i) The price elasticity of demand at $P = 4$ (3 marks)
- ii) At what level of Q is ATC at a minimum? (3 marks)

c) A production function is given by:

$$Q = 3L - \frac{1}{2}L^2$$

Where Q is output and L is labour input

i) Determine the average product of labour

ii) What is the corresponding marginal product of labour? (4 marks)

d) Find the $MRTS_{L,K}$ given the following production function:

$$Q = L^{1/4} K^{3/4} \quad (2 \text{ marks})$$

e) A two commodity market model is defined by the following:

$$Q_{d1} = 4 - P_1 + \frac{1}{2} P_2$$

$$Q_{d2} = 10 + P_1 - P_2$$

$$Q_{s1} = -3 + 4P_1$$

$$Q_{s2} = -18 + 4P_2$$

Determine equilibrium prices and quantities for the two commodities (5 marks)

QUESTION FOUR (20 MARKS)

a) Consider a demand function given by (3 marks)

$$Q = \frac{3}{P} \text{ determine the following:}$$

i) The $E_{Q,P}$ which is the price elasticity of demand.

ii) Show that the relationship between marginal revenue and elasticity of demand is valid

(4 marks)

$$MR = P \left\{ 1 + \frac{1}{E_{Q,P}} \right\}$$

b) The marginal propensity to consume for an economy is 0.72. Determine the savings function for the economy if $Y = 0$ when savings $S = -5$. (3 marks)

c) The demand and total cost functions facing a firm are:

$$P_1 = 26 - 3Q_1 - Q_2$$

$$P_2 = 33 - Q_1 - 2Q_2^2$$

$$TC = Q_1^2 + Q_1Q_2 + 2Q_2^2$$

Find:

- i) The average revenue functions for commodities Q_1 and Q_2 , (2 marks)
- ii) The total revenue (2 marks)
- iii) The average cost function with respect to Q_1 and Q_2 (2 marks)
- iv) The profit function (2 marks)
- v) The profit maximising quantity (2 marks)

QUESTION FIVE (20 MARKS)

a) You are given the following national Income model

$$Y = C + I + G + x - m \dots\dots\dots (1)$$

$$C = C_0 + C_1 (Y - T) \dots\dots\dots (2)$$

$$T = t_0 + t_1 Y \dots\dots\dots (3)$$

$$I = I_0 \dots\dots\dots (4)$$

$$G = G_0 \dots\dots\dots (5)$$

$$X = x_0 \dots\dots\dots (6)$$

$$M = M_0 + m_1 Y \dots\dots\dots (7)$$

- i) Plot equations 6 and 7 on the same diagram and label curves as X and M, respectively. Indicate the equilibrium of the system on the diagram, (3 marks)
 - ii) Find the equilibrium income for the system (4 marks)
 - iii) What is the corresponding equilibrium consumption and import. (4 marks)
- b) Consider a demand function given by $Q = 49/p^2$. Determine the consumer's surplus for a market price of $P_e = 4$. (4 marks)
- c) Find the critical values of following function and determine whether or not the values give rise to a local maximum or minimum (5 marks)

$$y = x^3 - 6x^2 + 9x + 15$$