



MACHAKOS UNIVERSITY

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

Examinations for the degree of

Bachelor of education (science & arts), Bachelor of Science (statistics, mathematics)

SMA 102: BASIC MATHS

Date:

Time: 2 Hours

Instructions to Candidates

Answer question ONE and any other TWO questions

QUESTION 1: 30 MARKS (COMPULSORY)

- a) In a recent survey of 400 students in a school 100 were listed as smokers and 150 as chewers of gum; 75 were listed as both smokers and gum chewers. Find how many students were neither smokers nor chewers of gum. (4 Marks).
- b) Solve the inequality: $(x - 6)(x + 1) > 2x - 12$. (5 Marks).
- c) When $x^3 - x^2 + ax + b$ is divided by $(x - 1)$ the remainder is -8 and when the same expression is divided by $(x + 1)$ the remainder zero. Calculate the values of a and b , and factorize the expression completely. (5 Marks)

- d) Determine the product of the complex numbers $Z_1 = \frac{1}{2} + \frac{1}{2}i$ and $Z_2 = -\frac{1}{4} - \frac{3}{4}i$, then show on an argand diagram the radius vector corresponding to this product. **(5 Marks).**
- e) A company wants to select a team of 8 workers for its repair operations. There are only 6 women and 15 men with this expertise. In how many ways can the selection be done if at least five men are to be in the team? **(4 Marks)**
- f) Determine whether the following is a tautology, a contradiction or neither by constructing a truth table. $(P \Rightarrow Q) \wedge (\sim P \vee Q)$ **(4 Marks).**
- g) Use mathematical induction to prove that: $1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ **(3 Marks)**

QUESTION 2 (20 MARKS)

- a) Describe each of the following terms as used in set theory giving an example in each case:
- Symmetric difference sets A and B . **(3 Marks)**
 - The power set of a set. **(3 Marks)**
 - Product of sets. **(3 Marks)**
- b) In a survey of the dinning preferences of 110 university resident students at the end of a semester, the following facts were discovered about *Adams Hotel* (A), *Brothers Pizza* (B), and *University Cafeteria* (C).
- 30 liked A but not B .
 - 21 liked A only.
 - 63 liked A
 - 58 liked B
 - 25 liked B and A but not C
 - 18 liked B and C .
 - 25 liked home cooking (i.e. disliked all three)
- Draw a Venn diagram that represents this survey and label all the areas. **(3 Marks)**
 - How many liked A or C **(3 Marks)**
 - How many liked all three? **(3 Marks)**
 - How many liked only C ? **(2 Marks)**

QUESTION 3 (20 MARKS)

- a) Express with real denominator: $Z = \frac{1}{2+3i} + \frac{1}{2-3i}$. **(4 Marks)**

- b) The roots of the equation $z^2 - z + 1 = 0$ are $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$. Find r_1 , r_2 , θ_1 , and θ_2 (6

Marks)

- c) Find the square roots of $-5 + 12i$ by equating the real and imaginary parts of

$$(a + bi)^2 = -5 + 12i \quad (5 \text{ Marks})$$

- d) Determine the locus given by $Z\bar{Z} + 2(Z + \bar{Z}) = 0$, where $Z = x + iy$ and $\bar{Z}$ is its complex conjugate. (5 Marks)

QUESTION 4 (20 MARKS)

- a) Find the number of distinguishable ways of arranging the letters in the word.

MISCELLANEOUS (4

Marks)

- b) Show that $(3x + 1)$ is a factor of the expression $3x^3 + 4x^2 + 7x + 2$. (4

Marks)

- c) Find the values of a and b if $(x - 2)$ and $(x + 1)$ are both factors of $ax^3 + 3x^2 - 9x + b$ and state the third factor of the expression. (6

Marks)

- d) Determine the first four terms of the expansion of $(1 + x)^{21}$ in ascending powers of x . Hence find the coefficient of x^3 in the expansion of $(2 - 3x)(1 + x)^{21}$. (6

Marks

QUESTIONS 5 (20 MARKS)

- a) Use Element argument method or Formal proof to show that $A \setminus B = A \cap B^c$. (4 Marks)

- b) Prove by mathematical induction that:

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + \cdots + [n(n + 1)] = \frac{1}{3}n(n + 1)(n + 2) \quad (4 \text{ Marks})$$

- c) Simplify: $(-1 + i)(1 - i\sqrt{3})$, leaving your answer in the form $z = r(\cos\theta + i\sin\theta)$, where θ is the principal value of the argument. Deduce the value of $\sqrt[3]{z^2}$.

(6 Marks)

d) Find the range of values of x for which $3x + 4 < x^2 - 6 < 9 - 2x$.

(6

Marks)