



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE IN (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE IN (CIVIL ENGINEERING)

ECU 200: ENGINEERING MATHEMATICS V

DATE: 22/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Attempt Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) State the fundamental theorem of Integral Calculus. (1 mark)
- b) Find the following integrals:
- i. $\int \frac{3x+1}{(x-1)(x^2+1)} dx$ (4 marks)
- ii. $\int_0^{\pi} 6x \sin(x^2) dx$ (4 marks)
- iii. $\int \sin^4 t \cos t dt$ (3 marks)
- c) Find the number c that satisfies the mean value theorem of integral calculus for the function $y = \frac{1}{2x+1}$ over the interval $[0, 1]$. (3 marks)
- d) Determine the double integral $\iint x^2 y dy dx$. (2 marks)
- e) Given $Ae^x + Be^{-x} = 4 \cosh x - 5 \sinh x$. Determine the values of A and B . (3 marks)

- f) Evaluate the area bounded by the curve $y = \sin x$, the x-axis and the lines $x=0$ and $x = \frac{3\pi}{2}$.
(4 marks)
- g) Use Euler's method to obtain a numerical solution of the differential equation $\frac{dy}{dx} = 3(1+x) - y$ given the initial conditions that $x=1$ when $y=4$, for the range $x=1.0$ to $x=2.0$ with intervals of 0.2.
(6 marks)

QUESTION TWO (20 MARKS)

- a) Given $y = \frac{x}{\cos x}$, find $\frac{dy}{dx}$ and hence determine $\int (\sec x + x \tan x \sec x) dx$.
(3 marks)
- b) Evaluate the volume of revolution generated by rotating about the x-axis, the region bounded by the function $f(x) = e^x$ and the lines $x=0$ and $x = \ln 2$.
(3 marks)
- c) Express $\frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2}$ in partial fractions and use your result to find the integral $\int \frac{x^3 + 2x^2 + x - 1}{x^2 + x - 2} dx$.
(6 marks)
- d) Using trapezoidal rule estimate $\int_1^4 \ln x dx$. Convert to four decimal places and take $r = 6$.
(5 marks)
- e) Prove the hyperbolic identity $\cosh^2 x - \sinh^2 x = 1$.
(3 marks)

QUESTION THREE (20 MARKS)

- a) Calculate the length of the arc of the curve $y = \sqrt{x}$ between $x=0$ and $x=3$.
(4 marks)
- b) Use the substitution $t = \tan \frac{x}{2}$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{4 + 5 \cos x} dx$.
(5 marks)
- c) Determine the value of the integral $\int_0^{\frac{4}{3}} \frac{1}{\sqrt{16 - 9x^2}} dx$.
(4 marks)
- d) Use tabular integration to evaluate $\int x^4 e^{-x} dx$.
(3 marks)

- e) A chain hangs so that its shape is of the form $y = 56 \cosh\left(\frac{x}{56}\right)$. Determine, correct to 4 significant figures:

- i. The value of y when x is 35. (2 marks)
- ii. The value of x when y is 62.35. (2 marks)

QUESTION FOUR (20 MARKS)

- a) A particle starts in motion with velocity of $10m/s^2$ and its acceleration after time t is given by $a = -2t + 3$. Find the distance travelled when the particle first comes to rest. (7 marks)
- b) Evaluate the double integral $\int_0^{\pi} \int_2^4 x \sin y \, dx \, dy$ (3 marks)
- c) Use integration by parts to find $\int x^2 (\log_e x)^2 \, dx$ (4 marks)
- d) Determine the power series for $2 \cosh\left(\frac{\theta}{2}\right) - \sinh 2\theta$ as far as the term θ^5 . (3 marks)
- e) Find the area enclosed by the curve $y = x(x-1)(x-2)$ and the axis between $x=0$ and $x=4$. (3 marks)

QUESTION FIVE (20 MARKS)

- a) A yeast culture is growing at a rate of $\frac{d\mu}{dt} = 0.3e^{0.2t}$, where t is the time in hours, $\mu(t)$ is the mass in grams. At time $t=0$ the mass was 0.1 grams.
 - i. Use Simpson's Rule with $r=6$ to estimate the mass after 3 hours. (5 marks)
 - ii. Estimate the absolute value of the maximum error that can occur when growing the culture after 3 hours. (3 marks)
- b) Evaluate $\int \frac{1}{\sqrt{9-4x^2}} \, dx$. (4 marks)
- c) The region bounded by the curve $y = x^2$ and the line $y = x$ is rotated about the y -axis. Calculate the volume of the solid of revolution generated. (4 marks)
- d) Find the average value f_{avg} of the function $y = 2 + \sin^2 x$ over the interval $\left[\frac{\pi}{2}, \pi\right]$. Hence find a value such that $f(c) = f_{avg}$. (4 marks)