MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 261: PROBABILITY AND STATISTICS II

DATE: 2/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTION: *Answer Question **ONE** which is compulsory*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the $E(x_1 + x_2)$ from the Bivariate table below. (3 marks)

		x_2	
		0	1
x_1	0	$\frac{1}{6}$	0
	1	⅓	⅓
	2	0	$\frac{1}{6}$

b) If the joint probability density function of X and Y is given by:

$$f(x, y) = 3y, 0 \leq y \leq 1, 0 < x < 1.\text{Determine } \mathbf{F}\left(\tfrac{1}{3}, \tfrac{1}{2}\right)$$ (6 marks)

c) The joint probability density function for x and y is given by $f(x, y) = \frac{1}{21}(x + y)$

x = 1, 2, 3.
y = 1, 2

(i) Determine marginal distribution of *x* and *y* (6 marks)
(ii) Prove for independence (4 marks)

d) Show that E(X+Y)=E(X)+E(Y) if X and Y are independent continuous random variables (5 marks)

e) Given that $f(x,y) = \begin{cases} 2x, 0 < x, y < 1 \\ 0 \quad otherwise \end{cases}$

Determine $p(0 < x < ⅓, ½ < y < 1)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Given y_1 and y_2 have the joint probability density function (pdf) given by

$$f(y_1, y_2) = \begin{cases} K(1 - y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Determine the value of the constant K. (4 marks)
- Determine $p(y_1 \le \frac{3}{4}, y_2 \ge \frac{1}{2})$ (4 marks)

b) If x and y have joint probability density function $f(x,y) = \frac{8x^2}{7y^3}$ for $1 \le x, y \le 2$

i) Determine the marginal distribution of x and y (8 marks)
ii) Determine the conditional density function of *y* given *X* =*x* (2 marks)
iii) Determine whether *x* and *y* are independent. (2 marks)

QUESTION THREE (20 MARKS)

a) Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$f(x,y) = \frac{yx^{y-1}}{3}$,$0 < x < 1, y \in$ 1,2,3.Determine

(i) $f_y(y)$ (5 marks)
(ii) $E(X)$ (5 marks)

b) Consider the Bivariate table below

		y = 0	y = 1
x	0	1/8	0
	1	2/8	1/8
	2	1/8	2/8
	3	0	1/8

i) Determine $E(x)$ (2 marks)
ii) Determine the covariance of x and y (4 marks)
iii) The marginal probability mass function of y (4 marks)

QUESTION FOUR (20 MARKS)

a) Let $y_1 \sim B(n_1, p)$ and $y_2 \sim B(n_2, p)$. Determine the moment generating function of $y_1 + y_2$ given that y_1 and y_2 are independent binomial random variables. (6 marks)

b) The joint pdf of x and y is given by that $f(x,y) = \begin{cases} 6x^2y, & 0 < \mathrm{x} < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

Determine: (i) μ_x *and* μ_y (4 marks)
(ii) d_x^2 *and* d_y^2 (4 marks)
(iii) The correlation coefficient between x and y (6 marks)

QUESTION FIVE (20 MARKS)

a) Let y be a continuous random variable with a pdf defined by that

$$f(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Using the change of variable technique determine the density of$x = 8y^3$. (8 marks)

b) Given that E(y/x)=$\propto + \beta x$, where E denotes expectation, x and y denotes random variables and $\propto$ and β are constants. Show that:

$$\beta = \frac{E(xy) - E(x)E(y)}{E(x^2) - (E(x))^2} = \frac{cov(x,y)}{var(x)}$$ (12 marks)