MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (COMPUTER AND MATHEMATICS)

BACHELOR OF SCIENCE (COMPUTER AND STATISTICS)

BACHELOR OF EDUCATION (SPECIAL EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 432: PARTIAL DIFFERENTIAL EQUATION I

DATE: 3/8/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Solve the differential equation $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$ (5 marks)

b) Determine the orthogonal trajectories on the conicoid $(x+y)z = 1$ of the conics in which it is cut by the system of planes $x - y + z = k$, where k is a parameter. (5 marks)

c) Solve the Lagrange's equation $y^2 p - xyq = x(z - 2y)$ (5 marks)

d) Work out the characteristics and the corresponding transport equations of the system

$$xyu_x - v_y = 0$$
$$xu_y - v_x = 0$$

(5 marks)

e) Determine the complete integral of $z = px + qy + p^2 + q^2$ using Charpit's method (5 marks)

f) Eliminate the arbitrary function f from the equation $f\left(x^2 + y^2 + z^2, z^2 - 2xy\right) = 0$ (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the partial differential equation $p^2 + q^2 - 2pq \tanh 2y = \sec h^2 2y$;

i.) Write its Charpit's auxiliary equations (2 marks)

ii.) Solve the p.d.e by considering its Charpit's auxiliary equations. (8 marks)

b) Using Jacobi's method determine a complete integral of $p_1 x_1 + p_2 x_2 = p_3^2$ (10 marks)

QUESTION THREE (20 MARKS)

a) Show that $(2x + y^2 + 2xz)dx + 2xydy + x^2 dz = 0$ is integrable (5 marks)

b) Solve;

i.) $\dfrac{dx}{x^2} = \dfrac{dy}{y^2} = \dfrac{dz}{nxy}$ (7 marks)

ii.) $\dfrac{dx}{y^2(x-y)} = \dfrac{dy}{-x^2(x-y)} = \dfrac{dz}{z(x^2 - y^2)}$ (8 marks)

QUESTION FOUR (20 MARKS)

a) Consider the partial differential equation $z = px + qy + p + q - pq$ with a complete integral of the form $z = ax + by + p + a + b - ab$; where a *and* b are arbitrary constants. Determine a general solution by finding the envelope of those planes that pass through the origin. (5 marks)

b) Solve the semi-linear equation $x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x + y)z$ (7 marks)

c) Determine the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = c$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the integral surface of the quasi-linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2 - y^2 z)$ which contains the straight line $x+y=0$, $z=1$

(8 marks)

b) in the one-dimensional unsteady flow of compressible fluid, the velocity u *and* c where $c^2 = \frac{dp}{d\rho}$, p is pressure and ρ is density satisfying the following equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + 2c\frac{\partial c}{\partial x} = 0$$

$$2\frac{\partial c}{\partial t} + 2u\frac{\partial c}{\partial x} + c\frac{\partial u}{\partial x} = 0$$

Prove that the characteristics are given by the differential equation $dx = (u \pm c)dt$ and that on the characteristic $dx = (u+c)dt$, $u+2c$ is a constant. (12 marks)