# MACHAKOS UNIVERSITY

**University Examinations 2019/2020 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR**

**BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE**

**BACHELOR OF SCIENCE IN MATHEMATICS**

**BACHELOR OF EDUCATION**

**SMA 400: TOPOLOGY**

**DATE:** **TIME:**

**INSTRUCTIONS**

**Answer ALL the questions in Section A and ANY TWO Questions in Section B**

**SECTION A**

**QUESTION ONE 30 MARKS (COMPULSORY)**

a) Let $X = \{1,2,3,4\}$ and $\rho = \{\emptyset,\ X,\ \{1\},\ \{2\},\ \{1,2\},\ \{1,2,3\},\ \{2,3\}\ \}$. Let $E = \{1, 2,3\}$. Find the set of all limit points of E. (4 marks)

b) Give the definition of a topological space. (4 marks)

c) Let $(X, \rho)$ be a topological space show that if $A, B \subset X$ and $A \subset B$ then $A^d \subset B^d$ (4 marks)

d) Let X be a non void set $\rho_d$ be a discrete topology in X. If $E \subset X$ find $E^d$ (the set of all limit points of E). (5 marks)

e) Let $X = \{i, j, k, l\}$. Let $S = \{\{i\}, \{j, k\}, \{k, l\}\}$ be a collection of subsets of $X$. Give the topology generated by the set $S$. (5 marks)

f) Let $X = \{i, j,\ k,\ l,\ m\}$ and $\rho = \{\emptyset,\ X,\ \{i\},\ \{j\},\ \{i, j\},\ \{k\},\ \{i, j, k\}, \{i, k\},\ \{j, k\}\ \}$.

g) Let $E = \{a, b, c\}$. Find:

i) The closure of E

ii) The interior of E (4 marks)

**QUESTION TWO (20 MARKS)**

a) Consider R with the usual topology and let $Y = (0,1) \cup \{2\}$ if $E = (\frac{1}{2}, 1)$ find the closure of E in the subspace Y. (6 marks)

b) Find the topology generated in R by the class of all closed sub-intervals of the type $(b, b + 1)$ . (5 marks)

c) Find the topology generated by the sub-basis $\beta = \{\{a\}, \{a, c, d\}, \{b, c\}, \{c\}\}$, where $X = \{a, b, c, d\}$ (5 marks)

d) Let $X$ be a non-empty set. Define the following topologies

i. Trivial topology

ii. Discrete topology (2 marks)

e) Define the neighborhood of a point $p \in X$ where $X$ is a topological space. (2 marks)

**QUESTION THREE (20 MARKS)**

a) Given any collection of subsets of a nonvoid set X. Will this collection serve as a base for the topology? (5 marks)

b) If $N_1$ $and$ $N_2$ are two neighborhoods of x, show that $N_1 \cap N_2$ is also a neighborhood of x. . (5 marks)

c) Consider the topology $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,3,4,5\}\}$ on $X = \{1,2,3,4,5\}$ and the subset

d) $A = \{b, d, e\}$ of $X$. Compute showing working out:

i. $Int(A)$ (3 marks)

ii. $Ext(A)$ (3 marks)

iii. $B'dary(A)$ (4 marks)

**QUESTION FOUR (20 MARKS)**

Let $(X, \rho)$ be a topological space show that; If $y \in F^d$ then $y \in (F - \{y\})^d$ . (5 marks)

a) Let $(X, \tau)$ be a topological space. Define a base and sub-base for the topology $\tau$ on $X$. (4 marks)

b) Let $X = \{x, y, z\}$ and $\tau = \{\{x, y\}, \{y, z\}, \emptyset, \{y\}, \{x, y, z\}\}$. Give the base and sub-base for the topology $\tau$ on $X$. (4 marks)

c) Given a subset $A$ of a topological space, define what it means for $p$ to be a limit point of $A$. (2 marks)

d) Consider the discrete topology $\tau$ on $X = \{1,2,3,4\}$. Find the sub-base and the bases for $\tau$. (4 marks)

**QUESTION FIVE (20 MARKS)**

a) Let $(X, \partial)$ be a metric space and A $\subset X$. Then prove that if P is a limit point of A every neighborhood of P contains infinitely many points distinct from P. (4 marks)

b) Let $X = \{a, b, c, d, e\}$ and $J = \left\{\emptyset, x\{a\}\{b, c\}\{b, c, d\}\{a, b, c\}\{a, d\}\{d\}\{a, b, c, d\}\right\}$ let $Y = \{a, b, c, d\}$,

   i) find the relative topology on Y

   ii) Define relative topology

   iii) Find the closed set in relative topology (6 marks)

c) Define hereditary as used in topological spaces (2 marks)

d) $T_0$ and $T_1$ spaces are hereditary prove (4 marks)

e) Show that every metric space is a $T_2$ space. (4 marks)