MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SAC 401: SURVIVAL MODELS

DATE: 17TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE (30 MARKS)

a) Let T denote the complete future lifetime of a newborn baby. The hazard rate for mortality at age x is defined to be:

$$\mu_x = \lim_{h-0+} \frac{1}{h} * P(T \leq x + h | T > x)$$

Now let Tx denote the complete future lifetime of a life aged exactly x who is subject to the same mortality rates. Show that, when t > 0:

$$\mu_{x+t} = \lim_{h-0+} \frac{1}{h} * P(Tx \leq t + h | Tx > t)$$ (5 marks)

b) Mortality of a group of lives is assumed to follow Gompertz' law. Calculate μ_x for a 30-year old and a 70-year old, given that μ_x is 0.003 for a 50-year-old and 0.01 for a 60-year old. (5 marks)

c) A cox proportional hazards model was estimated assess the effect on survival of a person's gender and his or her self-esteem. (Measured on a three – point scale as "low"," medium" or "high"). The baseline category was males with "low" self-esteem. Write down the equation of the model using algebraic symbols to represent variables and parameters and defining all the symbols that you use. (5 marks)

d) The main activity of a certain charity is training and providing guide dogs for blind people. The charity wishes to estimate the lifetime of the dogs it provides, and has therefore attempted to keep track of 100 dogs that were all born in 1990. Of these 100 dogs, 81 remained in service on reaching age 10 years. Of the remaining dogs, some died and some left active guide service for other reasons.

The ages of the deaths and other exits were recorded as follows:

Age (years)	**Number of deaths**	**Number of other exits**
0.25		2
0.5	1	
1		3
2.25		3
3.5		2
4	2	
4.5		1
6	1	
7.5		2
9		2

Calculate the Nelson-Aalen estimate of the integrated hazard for these guide dogs. (5 marks)

e) Show that mixing Poisson distribution with parameter ɵ and random variable x (prior) with exponential distributions with parameter ʎ yields geometric distribution. (5 marks)

f) The "Very-ruthless Management Consultancy Company" pays very high wages but also has a very high failure rate, both from sackings and through people leaving. A life table for a typical new recruit (with durations measured in years) would be:

Duration	**No of lives**
0	100,000
1	72,000
2	51,000
3	36,000
4	24,000
5	15,000
6	10,000
7	6,000

8	2,500
9	0

75 graduates started working at the company on 1 September this year. Calculate the following:

i) The expected number of complete years that a graduate will complete with the company. (2 marks)

ii) A graduate's expected "lifetime" with the company. (3 marks)

QUESTION TWO (20 MARKS)

a) Describe the essential feature of a proportional hazards model. (4 marks)

b) A study was made of the impact of drinking beer on men aged 60 years and over. A sample of men was followed from their 60th birthdays until they died, or left the study for other reasons. The baseline hazard of death, μ, was assumed to be constant, and a proportional hazards model was estimated with a single covariate: the average daily beer intake in standard-sized glasses consumed, x. The equation of the model is:

$h(t) = \mu \exp(\beta x)$

where h(t) is the hazard of death at age 60 + t.

The estimated value of μ is 0.03, and the estimated value of β is 0.2.

(i) Explain how μ and β should be interpreted, in the context of this model. (5 marks)

(ii) Calculate the estimated hazard of death of a man aged exactly 62 years who drinks two glasses of beer a day. (2 marks)

(iii) A man is aged exactly 60 years and drinks three glasses of beer a day.

- Calculate the estimated probability that this man will still be alive in 10 years' time. (2 marks)
- Calculate the expectation of life at age 60 years for this man. (2 marks)

(iv) Another man is aged exactly 60 years. He drinks beer only in his local bar. He drinks all the beer he buys and is expected to continue drinking the same amount of beer every day until he dies. The owner of the bar is interested in selling as much beer as possible.

Determine the average number of glasses of beer a day the owner must sell the man in order to maximize the total amount of beer the man buys over his remaining lifetime.

(5 marks)

QUESTION THREE (20 MARKS)

Kamau specializing in the baking of bread uses a disposable electric kitchen implement. The *implement* is rather unreliable and often breaks down, so the Kamau is in the habit of replacing the implement in use at a given time, shortly before an important social function or after making the 1,000th bread with that implement.

The following times until mechanical failure (no asterisk) or replacement whilst in working order (asterisk) were observed (measured in days of use):

22, 18, 11*, 3*, 26, 14*, 10, 23, 2*, 27, 15*, 20, 9, 16, 10*, 23, 15, 15, 4*, 22

a) What is the difference between an estimate and an estimator? (2 marks)

b) Calculate the Nelson-Aalen estimate of $F(t)$ for the *implement*. (9 marks)

c) Using Greenwood's formula, estimate var [(F(16))]for the *implement* data. Explain how you would use this figure to calculate a confidence interval. (9 marks)

QUESTION FOUR (20 MARKS)

a) Below is an extract from English life Table 15 (Males).

Age x	l_x
58	88,792
62	84,173

i) Estimate l_{60} under each of the following assumptions:

- A uniform distribution of deaths between exact ages 58 and 62 years and;
- A constant force of mortality between exact ages 58 and 62 years. (5marks)

ii) ii. Find the actual value of l_{60} in the tables and hence comment on the relative validity of the 2 assumptions you used in part (i). (4marks)

b) Express force of mortality, μ_x in terms of age (x + t) of an individual member included in the study. (7marks)

c) Given that ${}_tp_x = (\frac{1+y}{1+y+p})^3$ for $p \geq 0$

Calculate life expectancy of a person age y = 41 (4 marks)

QUESTION FIVE (20 MARKS)

a) A chef specialising in the manufacture of fluffy meringues uses a Whiskmatic disposable electric kitchen implement. The Whiskmatic is rather unreliable and often breaks down, so the chef is in the habit of replacing the implement in use at a given time, shortly before an important social function or after making the 1,000th fluffy meringue with that implement. The following times until mechanical failure (no asterisk) or replacement whilst in working order (asterisk) were observed (measured in days of use): 17, 13, 15*, 7*, 21, 18*, 5, 18, 6*, 22, 19*, 15, 4, 11, 14*, 18, 10, 10, 8*, 17

Define N, m, k, t_j , d_j , c_j and n_j for these data, assuming that censoring occurs just after the failures were observed. (4 marks)

b) You have been asked to advise a sports magazine, as a consultant statistician. You have been asked to investigate the hypothesis that football managers in the Italian "Serie A", are dismissed more quickly than those in the English premier league. Each league has twenty teams, each with one manager. During the season the following events happened (at the end of the months indicated) to the twenty managers who started:

MONTH	ITALY	ENGLAND
1	One dismissed	One died
3		One left of his own accord
5	One left of his own accord	One dismissed
6	Two dismissed	One left of her own accord
8	One died	Two dismissed

11	Two dismissed	One dismissed

Hence there were thirteen of the original twenty managers still employed by the same club at the end of the season, for each of the two leagues.

(i) Calculate the Kaplan-Meier estimates and their approximate variances for each league separately. (12 marks)

(ii) (ii) Comment on the hypothesis that Italian managers are dismissed more quickly than those in England. (4 marks)