



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

SMA335: ORDINARY DIFFERENTIAL EQUATION 1

DATE: 12/8/2021

TIME: 8:30 – 10:30 AM

INSTRUCTIONS:

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (COMPULSORY)(30MARKS)

- a) Determine the differential equation associated with the primitive $xy = Ae^x + Be^{-x} + x^2$ (4 marks)
- b) Solve the differential equation $\frac{dy}{dx} = x^2 - 2x + 2$ (4 marks)
- c) Determine the orthogonal trajectories of the family of curves $x^2 + y^2 = cx$. c is an arbitrary constant. (4 marks)
- d) Determine a particular solution to the second order differential equation $(D^2 - 1)y = xe^{3x}$ (4 marks)

- e) Calculate the integrating factor for the equation

$$\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)dy = 0 \quad (4 \text{ marks})$$

- f) Transform the equation below to its homogeneous form

$$(x + y)(dx - dy) = dx + dy \quad (5 \text{ marks})$$

- g) Solve the Bernoulli's equation $x \frac{dy}{dx} + y = y^2 \log x$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Define the 2^{nd} order linear differential equation with variable coefficients (2 marks)

- b) Solve the differential equations;

i.) $\frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 9y = 40 \sin 5x$ (7 marks)

ii.) $\frac{d^4 y}{dx^4} - y = x^2 \sin x$ (11 marks)

QUESTION THREE (20 MARKS)

- a) Consider the equation $\cos x (\cos x - \sin a \sin y)dx + \cos y (\cos y - \sin a \sin x)dy = 0$

i.) Show that it's exact in x and y (3 marks)

ii.) Solve the differential equation (5 marks)

- b) An RC circuit has an $e.m.f$ given (in volts) by $400 \cos 2t$, a resistance of 100 Ohms , and a capacitance of 10^{-1} farad . Initially there is no charge on the capacitor. Determine the current in the circuit at any time t . (12 marks)

QUESTION FOUR (20 MARKS)

- a) Consider the differential equation: $(x - y + 3)dx - (2x - 2y + 5)dy = 0$. Solve the differential equation by the method of reduction to homogeneous form.

(8 marks)

- b) Solve the differential equation with variable coefficients below:

$$x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 13y = e^{\log x} \quad (12 \text{ marks})$$

QUESTION FIVE (20 MARKS)

- a) A person deposits *kshs* 8,000 in an account that accrues interest compounded continuously. The interest rates are at a constant of 10% for the first four years and a constant of 12% for the next three years. Assuming there is no additional deposits or withdrawals. Determine the balance in his account after the seven years. (12 marks)

- b) Solve $x dx + y dy + \frac{xdy - ydx}{x^2 + y^2} = 0$ (8 marks)