MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

SMA 361: THEORY OF ESTIMATION

DATE: 7/12/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ

$$t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

$$t_2 = \frac{X_1 + X_2}{2} + X_3$$

$$t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

Where λ is such that t_3 is an unbiased estimator of μ

i. Determine λ (4 marks)

ii. Show that $t_1 and\ t_2$ are unbiased (4 marks)

iii. State giving reasons the estimators which is more efficient among $t_1, t_2, and\ t_3$ (4 marks)

b) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, x=0,1,….

Use the factorization theorem to show that $T = \sum x_i$ is a sufficient estimator of μ (4 marks)

c) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of μ (5 marks)

d) $Let\ x_1, x_2, .. x_n$ be identically independent random variable with a finite variance δ^2 .Show that the sample variance $s^2 = \frac{1}{n-1}(\sum_1^n (x-\bar{x})^2 = \frac{1}{n-1}\{\sum_1^n x^2 - n\bar{x}^2\}$ (4 marks)

e) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ.

(5 marks)

QUESTION TWO (20 MARKS)

a) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{bmatrix} (\lambda+1)x^{\lambda}, & 0 < \mathrm{x} < 1,\ \lambda > 0 \\ 0 & elsewhere \end{bmatrix}$$

Obtain the moment estimator of λ (8 marks)

b) A random sample of size n is drawn from a normal population with (0,δ^2). Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (11 marks)

QUESTION THREE (20 MARKS)

Three objects M, N and P with weights$t_{1,} t_2\ and, t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$

QUESTION FOUR (20 MARKS)

a) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$

(10 marks)

b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ. (10 marks)

QUESTION FIVE (20 MARKS)

a) Given that 8.6,7.9,8.3,6.4,8.4,9.8,7.2,7.8,7.5 are observed values of a random variable which is distributed normally with mean 8 and unknown variance δ^2.Given that

$\chi^2_{\frac{\alpha}{2},n} = 19.2$ and $\chi^2_{1-\frac{\alpha}{2},n}$=2.70.construct a 95% confidence interval forδ^2 (10 marks)

b) $Let\ x_1, x_2, ..\ x_n$be a random sample of size n from a population whose pdf is given by

$f(x, \theta) = \theta e^{-\theta x}$, x> 0,

Determine the Cramer-Rao lower bound for θ. (10 marks)