MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 301: REAL ANALYSIS II

DATE: 19/12/2017 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY Two Questions in Section B

SECTION A (COMPULSORY)

QUESTION ONE (30 MARKS))

a) Define the following terms

i) Absolute convergence

ii) Lower darboux sum

iii) Upper darboux

iv) Conditional convergence

v) Radius of convergence (5 marks)

b) Find the radius of convergence of the following series $\sum \frac{1}{n^2} x^n$ in $\mathbb{R}$. (5 marks)

c) Prove that if (X, ρ) is a metric space and $f: (X, \rho) \to (\mathbb{R}, \partial)$ is continuous and if $a, b \in X$ where $f(a) > 0$ and $f(b) < 0$. Then there are neighbourhoods $N(a),\ N(b)$ of a, b respectively such that $f(x) > 0\ \forall\ x \in N(a)$ and $f(x) < 0\ \ \forall\ \ x \in N(b)$. (5 marks)

d) Let x_n *and* y_n be convergent sequence of elements of $\mathbb{C}$ with limits x and y respectively. Prove that $\lim_{n\to\infty}(x_n + y_n) \quad = x + y$ (5 marks)

e) Find the sum of the series

$$\sum_{n=0}^{\infty} 2\sin(\frac{\pi}{6})^n$$ (5 marks)

f) Prove that all the monotonic functions on bounded intervals are functions of bounded variations. (5 marks)

QUESTION TWO (20 MARKS)

a) Test the convergence or divergence of the series $\sum_{n\in N}\frac{1}{n^p}$ if

i) $p > 1$

ii) $p < 1$

iii) $p = 1$ (4 marks each)

b) Prove that if $\sum z_n$ is absolutely convergent then $\sum z_n$ is convergent the converse is not true. (5 marks)

c) Determine whether the following series diverges or converges

$\sum_{n=1}^{\infty}\frac{2n}{5n+4}$ (3 marks)

QUESTION THREE 20 MARKS

a) Let $f\colon \mathbb{R} \to \mathbb{R}$ defined by $(x) = \begin{cases} 1 \; if \; x \in \mathbb{R} \\ 0 \; if \; x \in \mathbb{R} \end{cases}$. Does the right and the left sided limit exist?

(6 marks)

b) Let $f(x) = \begin{cases} x + 4, & if \; 0 < x < 1 \\ x + 5 & if \quad x \geq 1 \end{cases}$. Is the function continuous? (6 marks)

c) If V is monotonic increasing on $[a, b]$ that is $x, y\epsilon\ [a, b]$ and $< y$, then prove that $v(x) \leq v(y)$. (7 marks)

QUESTION FOUR 20 MARKS

a) Prove that if $f \epsilon Bv[a,b]$ and $D = v - f$ then D is monotonic increasing on $[a,b]$. (7 marks)

b) Prove that if $f:[a,b] \rightarrow R$ be bounded and α increasing on $[a,b]$ if p_1 *and* p_2 are any 2 partitions of $[a,b]$ then $l(f,\alpha,p_1) \leq U(f,\alpha,p_2)$. (7 marks)

c) Prove that if $f \epsilon Bv[a,b]$ if and only if it can be expressed as the difference of two monotonic increasing functions on $[a,b]$. (6 marks)

QUESTION FIVE 20 MARKS

a) Let $f(x) = \begin{cases} 2x+1, & if\ x \in (0,1) \\ -4 & if\ \ x \in (1,2) \end{cases}$

And $\alpha(x) = \begin{cases} x+3, & if\ x \in (0,1) \\ 6-x & if\ \ x \in (1,2) \end{cases}$ is $f \in \mathcal{R}(\alpha)$ 0n (0,2). (7 marks)

b) Define f by f(x)= 1 $\forall\ x \epsilon [a,b]$ show that $f \epsilon R[a,b]$ such that $\int_a^b f(x)dx = b - a$. (7 marks)

c) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$,determine whether the series converges or diverges using the integral test. (6 marks)