



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 432: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 26/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

QUESTION ONE (30MARKS) COMPULSORY

- a) Under what conditions can the Lagrange equation $Pp + Qq = R$ be classified as: Linear, Semi-Linear and Quasi-Linear. (4 marks)
- b) Find a partial differential equation arising from $z = ax^2 + by^2 + ab$ (4 marks)
- c) Find the complete solution to $(y - z)\frac{\partial z}{\partial x} + (x - y)\frac{\partial z}{\partial y} = z - x$ (5 marks)
- d) Find the integral curves of the equations:
- i) $\frac{dx}{x + y} = \frac{dy}{x + y} = \frac{dz}{-(x + y + 2z)}$ (6 marks)
- ii) $y^2 zp - x^2 zq = x^2 y$ (5 marks)
- iii) $\frac{dx}{xz - y} = \frac{dy}{yz - x} = \frac{dz}{1 - z^2}$ (6 marks)

QUESTION TWO (20 MARKS)

- a) Show Pfaffian differential equation given by $z(x + y^2)dx + z(z + x^2) - xy(x + y)dz = 0$ is integrable. (7 marks)
- b) Find the solution of the equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (9 marks)
- c) Eliminate the arbitrary constant from $z = ax^2 - by^3 + 4ab$ (4 marks)

QUESTION THREE (20 MARKS)

- a) Given that $u \equiv u(x, y, z) = c_1$, $v \equiv v(x, y, z) = c_2$ are solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Show that

$F(u, v) = 0$ is a general solution of the Lagrange's equation. (10 marks)

- b) Find the integral curves of the equation $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (10 marks)

QUESTION FOUR (20 MARKS)

- a) Consider a surface $F(x, y, z) = 0$ and let this surface intersect with the family of surfaces $G(x, y, z) = k$. Using Cramer's rule, derive the equations:

$$P' = RF_y - QF_z, Q' = PF_z - RF_x, R' = QF_x - PF_y \quad (10 \text{ marks})$$

- b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1, z = 1$ (10 marks)

QUESTION FIVE (20 MARKS)

- a) Determine the general solution to the P.D.E.

$$xp + yq = 3z \quad (6 \text{ marks})$$

- b) Eliminate the arbitrary function from $\phi(x + y + z, x^2 + y^2 + z^2) = 0$ (4 marks)

- c) Find the integral surface of the equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$ (10 marks)