MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 230: VECTOR ANALYSIS

DATE: 11/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

1) Answer Question ONE and any other TWO questions.
2) Mobile phones and any written material are prohibited in the examination room.
3) No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.
4) All answer booklets should be handed in at the end of the exam whether used or not.
5) Programmable calculators are prohibited

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given $a = 3\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{c} = -\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$. Find the magnitude of $2\boldsymbol{a} - 3\boldsymbol{b} - 5\boldsymbol{c}$. (4 marks)

b) Find the angle between vectors $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$. (5 marks)

c) Let $\boldsymbol{u} = \boldsymbol{i} - 5\boldsymbol{k}$ and $\boldsymbol{w} = 3\boldsymbol{i} - 2\boldsymbol{j} + 5k$ find $\boldsymbol{u} \times \boldsymbol{w}$. (3 marks)

d) Find the equation of a plane determined by the points $P_1(1,3,5), P_2(-2,1,4)$ and $P_3(3,-1,2)$. (5 marks)

e) Given that $\boldsymbol{A} = t^2\boldsymbol{i} - t\boldsymbol{j} + (2t+1)\boldsymbol{k}$ and $\boldsymbol{B} = (2t-3)\boldsymbol{i} + \boldsymbol{j} - t\boldsymbol{k}$, work out $\frac{d}{dt}(\boldsymbol{A} \cdot \boldsymbol{B})$ at $t = 1$. (5 marks)

f) If $\varphi = 5x^2y^2 - y^2z$. Find $\nabla\varphi$ at the point $(2,-1,4)$. (4 marks)

g) Find the directional derivative of $\varphi = x^2y^3 + 4xz^2$ at $(1,-2,-1)$ in the direction $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ (4 marks)

QUESTION TWO (20 MARKS)

a) Prove the following Frenet-Serret formulae

(i) $\frac{d\tilde{t}}{ds} = \chi\tilde{n}$ (3 marks)

(ii) $\frac{d\tilde{b}}{ds} = -\tau\tilde{n}$ (4 marks)

(iii) $\frac{d\tilde{n}}{ds} = \tau\tilde{b} - \chi\tilde{t}$ (3 marks)

b) Given the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Find

(i) the curvature χ (5 marks)

(ii) the torsion τ (5 marks)

QUESTION THREE (20 MARKS)

a) If $\tilde{A} = (3x^2 + 6y)\tilde{\imath} - 14yz\tilde{\jmath} + 20xz^2\tilde{k}$, evaluate $\int_c \tilde{A} \cdot d\tilde{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths,

(i) $x = t, y = t^2, z = t^3$. (5 marks)

(ii) the straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$ (7 marks)

(iii) the straight line joining $(0,0,0)$ and $(1,1,1)$. (3 marks)

b) Find the total work done in moving a particle in a force field given by $\tilde{F} = 3xy\tilde{\imath} - 5z\tilde{\jmath} + 10x\tilde{k}$ along $x = t^2 + 1$, $y = 2t^2$ $z = t^3$ from $t = 1$ to $t = 2$

(5 marks)

QUESTION FOUR (20 MARKS)

a)

(i) Show that $\tilde{F} = (2xy + z^3)\tilde{\imath} + x^2\tilde{\jmath} + 3xz^2\tilde{k}$ is a conservative force field. (5 marks)

(ii) Find the scalar potential. (5 marks)

(iii) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

(5 marks)

b) Prove that $curl\ grad\ \varphi = 0$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\iint_s \tilde{A}\cdot\tilde{n}ds$ where $\tilde{A} = 18z\tilde{\imath} - 12\tilde{\jmath} + 3y\tilde{k}$ and s is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. (10 marks)

b) Let $\tilde{F} = 2xz\tilde{\imath} - x\tilde{\jmath} + y^2\tilde{k}$. Evaluate $\iiint_v \tilde{F}dv$ where v is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2,\ z = 4$. (6 marks)

c) Use Greens theorem to evaluate $\oint_c xydx + x^2y^3dy$ where c is the triangle with vertices $(0,0), (1,0), (1,2)$ with positive orientation. (4 marks)