MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR BACHELOR OF SCIENCE (MATHEMATICS)

SMA 462: OPERATION RESEARCH III

DATE: 18/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTIONS : answer question one and any other two questions.

QUESTION ONE (30 MARKS)

a) State the areas of application of integer programming (3 marks)

b) Explain the necessity of goal programming (3 marks)

c) Construct the dual of the of the problem:

$$\textit{Minimize} \quad Z = x_2 + 3x_3$$

$$\textit{Subject to} \quad \begin{aligned} &2x_1 + x_2 \leq 3, \\ &x_1 + 2x_2 + 6x_3 \geq 5 \\ &-x_1 + x_2 + 2x_3 = 2 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

(4 marks)

d) Determine whether the function is convex, concave or neither

$$f(X) = x_1^2 + x_2^2 - 2x_1x_2$$

(4 marks)

e) Solve the NLPP using the Kuhn-Tucker conditions

$$\textit{Maximize} \quad Z = 4x_1 - x_1^3 + 2x_2$$

$$\textit{Subject to} \quad \begin{aligned} &x_1 + x_2 \leq 1 \\ &x_1, x_2 \geq 0 \end{aligned}$$

(5 marks)

f) Solve the NLPP using the Lagrangean multiplier

Maximize $Z = 5x_1 + x_2 - (x_1 - x_2)^2$

Subject to $x_1 + x_2 = 4$; $x_1, x_2 \geq 0$ (5 marks)

g) Mulley's Company is known for the manufacture of tables and chairs. There is a profit of Rs.200 per table and Rs.80 per chair. Production of table requires 5 hours of assembly and 3 hours in finishing. In order to produce a chair, the requirements are 3 hours for assembly and 2 hours of finishing. The company has 105 hours of assembly time and 65 hours of finishing. The company manager is interested to find out the optimal production of tables and chair so as to have maximum profit of Rs.4,000. Formulate a goal programming for this situation (6 marks)

QUESTION TWO (20 MARKS)

a) Solve the following NLPP using the Kuhn-Tucker conditions

Maximize $Z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

Subject to $2x_1 + 5x_2 \leq 98$; $x_1, x_2 \geq 0$ (8 marks)

b) Use duality to solve the following problem:

Minimize $Z = 0.7x_1 + 0.5x_3$

Subject to (12 marks)

$x_1 \leq 4,$

$x_2 \geq 6$

$x_1 + 2x_2 \geq 20$

$2x_1 + x_2 \geq 18$

$x_1, x_2 \geq 0$

QUESTION THREE (20 MARKS)

a) Define integer linear programming (2 marks)

b) Use Gomory's cutting plane algorithm to solve:

Maximize $Z = x_1 - x_2$

Subject to (18 marks)

$x_1 + 2x_2 \leq 4,$

$6x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0 - and - are - \text{int}\, egers$

QUESTION FOUR (20 MARKS)

A company manufactures two products, radio and transistors, which must be processed through assembly and finishing departments. Assembly has 90 hours available; finishing can handle 72 hours of the work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each transistor requires 3 hours in assembly and 6 hours in finishing. If a profit is Rs.120 per radio and Rs.90 per transistor, determine the best combinations of radios and transistors to realize of Rs.1500 and to meet the demand of 10 radios, obtain the optimal solution (20 marks)

QUESTION FIVE (20 MARKS)

a) Give Mathematical definitions of a concave and a convex (4 marks)

b) Explainthe nature of function $f(x) = 2x^3 - 3x^2$ (4 marks)

c) Consider the following function:

$f(X) = 5x_1 + 2x_2^2 + x_3^2 - 3x_3x_4 + 4x_4^2 + 2x_5^4 + x_5^2 + 3x_5x_6 + 6x_6^2 + 3x_6x_7 + x_7^2$, show that $f(X)$ is convex by expressing it as a sum of functions of or two variables and then proving that all the functions are convex (12 marks)