



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 231: DYNAMICS I

DATE: 24/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30MARKS)

- a) On turning a corner, a motorist rushing at $20m/s$ finds a child on the road $50m$ ahead. He instantly stops the engine and applies brakes so as to stop the car within $10m$ off the child. Determine;

- The retardation (3 marks)
- Time required to stop the car (2 marks)

- b) The torque $\vec{\Lambda}$ acting on a particle equals to time rate of change in angular momentum $\vec{\Omega}$. Show that;

$$\vec{\Lambda} = \frac{d\vec{\Omega}}{dt} \quad (4 \text{ marks})$$

- c) A plane has a rise of 5 in 12. A shot is projected with a velocity of $200m/s$ at an elevation of 30° . Calculate the range of the plane if;
- The shot is fired up the plane (3 marks)
 - The shot is fired down the plane (2 marks)

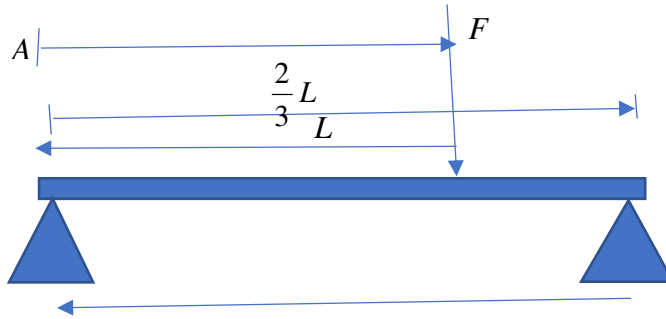
- d) A conservative force field defined by $\vec{F} = (y^2 z^3 - 6xz^2)\hat{i} + 2xyz^3\hat{j} + (3xy^2 z^2 - 6x^2 z)\hat{k}$ has a scalar function defined by $\phi = 3x^2 z^2 - xy^2 z^3$. Calculate the work done by the force field in moving a particle from the point $A(-2, 1, 3)$ to the point $B(1, -2, -1)$ (3 marks)
- e) Derive the equation $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ for the simple harmonic oscillator (5 marks)
- f) Two tanks sit side by side on the floor. The larger tank (52kg) is to the left of the smaller tank (34kg). A person pushes on the larger tank horizontally towards the right. The coefficient of static friction between the tanks and the floor is 0.35. Determine;
- The magnitude of the maximum force that the person can exert without moving any tank. (2 marks)
 - The force the larger tank exerts on the smaller tank. (2 marks)
- g) A bomb is released from an aeroplane flying at 800km/hour at a height of 1000m. Calculate the horizontal distance travelled by the bomb during the flight. (4 marks)

QUESTION TWO (20 MARKS)

- a.) A particle of mass m travelling in a straight line is supplied with constant power P and is subjected to resistance Mkv^2 , where v is its speed and k is a constant.
- Prove that the speed of the body cannot exceed a certain value. (6 marks)
 - Prove that if it starts from rest it acquires half the maximum speed after travelling a distance $\frac{1}{3}k \ln\left(\frac{8}{7}\right)$. (7 marks)
 - If the power is then cut off and an additional retarding force of constant value $\vec{F}$ is imposed. Calculate the subsequent time which elapses before the body comes to rest. (7 marks)

QUESTION THREE (20MARKS)

- a.) Three objects are arranged in a 2-dimensional coordinate system as; $A: 3kg(0.5m, 2.5m)$, $B: 3kg(3.5m, 2.5m)$, $C: 4kg(2m, 0.5m)$. Determine the center of mass of three objects. (6 marks)
- b.) Consider a simple supported beam with a heavy load 6,000N as below.



Determine the forces at the ends of the beams. (7 marks)

- c.) A particle of mass m moves along the x -axis under the influence of a conservative force field having potential $\phi(x)$. If the particle is located at the position x_1 and x_2 at the respective times t_1 and t_2 . Prove that if E is the total energy then;

$$t_2 - t_1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - \phi(x)}} \quad (7 \text{ marks})$$

QUESTION FOUR (20MARKS)

An object of mass 20kg moves with simple harmonic motion on the x -axis. Initially it is located at the distance 4m away from the origin ($x = 0$), has a velocity of 15m/s and acceleration of 100m/s^2 directed towards $x = 0$.

- Determine the position at any time t (7 marks)
- Calculate;
 - Amplitude (1 mark)
 - Period (2 marks)
 - Frequency (2 marks)
- Obtain the force on the object when $t = \frac{\pi}{10}\text{sec}$ (8 marks)

QUESTION FIVE (20 MARKS)

A batsman hits a cricket ball, giving it a velocity of 25m/s at an elevation of $\arctan \frac{4}{3}$. Calculate;

- the greatest height attained (5 marks)
- the range on level ground (5 marks)
- the time of flight (5 marks)
- height of the ball when it reaches the boundary which is 55m from the batsman. (5 marks)