MACHAKOS UNIVERSITY

University Examinations 2019/2020Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SCIENCE)

SPH 200: MECHANIC II

DATE: 28/11/2019 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer ***Question One*** *(compulsory) and any* ***TWO questions*** *from the rest of the questions.*

IMPORTANT CONSTANTS

Acceleration done to gravity g=9.81N/Kg

QUESTION ONE (30 MARKS)

a) Define the following terms

i. conservative force (2 marks)

ii. radius of gyration (2 marks)

b) Find the total work done in moving a particle in a force field $\vec{\mathrm{F}} = 3\mathrm{xy}\,\mathbf{i} - 5\mathrm{z}\,\mathbf{j} - 10\mathrm{x}\,\mathbf{k}$ along a curve with parametric equations $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 2$ to $t = 3$. (4 marks)

c) Suppose that a particle of constant mass m at times t_1 and t_2 is located at points P_1 and P_2 while moving with velocities v_1 and v_2 respectively. Prove that the work done from moving the particle from P_1 to P_2 is given by $W = \frac{1}{2}m(v_2^2 - v_1^2)$. (4 marks)

d) Determine the torque about the origin of a particle at a point (5, 2, 7) due to the following force fields

i. $\vec{\mathrm{F}} = 6\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ (2 marks)

ii. $\vec{\mathrm{F}} = -\mathbf{i} - 8\mathbf{j} + 5\,\mathbf{k}$ (2 marks)

e) A force field is defined by $\vec{F} = (y^2z^3 - 6xz^2)\mathbf{i} + (2xyz^3)\,\mathbf{j} + (3xy^2z^2 - 6x^2z)\,\mathbf{k}$,

i. show that $\vec{F}$ is conservative, (3 marks)

ii. find the scalar potential V of the force field, and (4 marks)

iii. calculate the work done in moving an object in the force field from point $P\ (1,3,2)$ to the point $Q\ (3,4,5)$. (3 marks)

f) Determine the expression of moment of inertia of a uniform rod of length $2l$ and mass m rotating about its center. (4 marks)

QUESTION TWO (20 MARKS)

a) State the following theorems of moment of inertia

i. Parallel axis theorem (2 marks)

ii. Perpendicular axis theorem (2 marks)

b) Find the moment of inertia of a hollow cylinder of length L, mass M inner radius R_1 and outer radius R_2, rotating about its central axis along its length. (4 marks)

c) A dumbbell consists of two uniform spheres of mass M_s and radius R joined by a thin rod of mass m, length L, and radius r (see Fig 2.1). What is the moment of inertia

i. about the center of mass and perpendicular to the rod (Axis A)? (4 marks)

ii. about an axis through one sphere and perpendicular to the rod (Axis B)? (4 marks)

iii. about an axis along the rod (Axis C)? (4 marks)

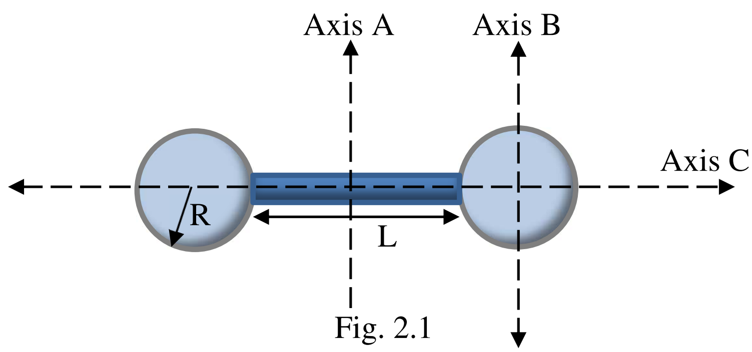

Fig. 2.1

QUESTION THREE (20 MARKS)

a) In a simple harmonic motion (SHM) for a mass and spring, the magnitude of the force is given by $-kx$, where k is the force constant and x is the displacement of the mass (which is making SHM) from its equilibrium position.

i. Find the differential equation for the above simple harmonic motion. (3 marks)

ii. Taking the general solution to be $x = A\cos(\omega t + \varphi)$, solve the above differential equation and find the value of ω in terms of k and m. (6 marks)

iii. What is its maximum displacement? (2 marks)

iv. Find the time period T for the SHM if $m = 3$ kg and $k = 75$ N/m. (3 marks)

b) Show that in a simple pendulum when the bob is given a small displacement it makes simple harmonic motion. (3 marks)

c) A 40g mass is attached to the lower end of a spring of negligible mass and oscillates at a frequency of 2 Hz, with amplitude of 8 cm. How fast is the system moving when it is 2 cm from the equilibrium? (3 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms

i. Moment of inertia (2 marks)

ii. Angular momentum (2 marks)

b) i Define angular momentum and state its mathematical expression. (2 marks)

ii Show that the torque $\vec{\tau}$ acting on a particle equals the time rate of change of its angular momentum $\frac{d\vec{L}}{dt}$, hence show the condition for the conservation of angular momentum. (5 marks)

c) A particle moves along a space curve defined by $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$. Find the magnitude of the acceleration at $t = 0$. (4 marks)

d) Show that the force field $\vec{\mathrm{F}} = (2x^3y^4 + x)\mathbf{i} + (2x^4y^3 + y)\mathbf{j}$ is conservative, hence evaluate $\int_c \vec{\mathbf{F}}\,.\,\mathbf{d}\vec{\mathrm{r}}$ given that $\mathbf{r}(t) = (t\cos\pi t - 1)\,\mathbf{i} + \left(\sin\frac{\pi t}{2}\right)\mathbf{j},\ 0 \le t \le 1.$ (5 marks)

QUESTION FIVE (20 MARKS)

a) State three factors that affect the magnitude of moment of inertia. (3 marks)

b) The moment of inertia of a solid cylinder of radius 0.3 m is $2.2\ \mathrm{kg\,m^2}$. It is set in motion from rest by applying a tangential force of 29.6 N with a rope wound around its circumference.

i. Calculate the angular acceleration of the cylinder. (4 marks)

ii. What would be the acceleration if a mass of 2 kg were hung from the end of the rope instead of the tangential force? (4 marks)

c) A particle P has a mass of 2 kg and a velocity $(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ m/s. Determine the angular momentum of P about a point O given that the displacement vectors of P and O from a fixed point are $(3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$ m and $(\mathbf{i} + 7\mathbf{j} - 9\mathbf{k})$ m, respectively. (4 marks)

d) A circular disc of mass m and radius r rolls on a table. If it has an angular frequency ω, show that its total energy is given by $E = \frac{3}{4}mr^2\omega^2$. (5 marks)