MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SST 102: DISCRETE MATHEMATICS

DATE: 19/01/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and any other two questions

QUESTION ONE - (30 MARKS)

a) Explain the meanings of the following terms

i. a *partition* of a set A, (2 marks)

ii. a *surjective function* from a set A to a set B, (2 marks)

iii. a *recurrence relation* (2 marks)

b) Given that $S = \{n \in \mathbb{Z}: 1000 < n \leq 3000\}$. Determine then number of elements of S that are divisible by 5, but by not by 3. Justify your answers clearly. (5 marks)

c) Consider the sets $A = \{3,45,201\}$ and $B = \{2,17,45,48,300\}$ and let R be a relation defined from A to B such that for any $a \in A$ and $b \in B$ then aRb if and only if a is greater than B or if a divides b. Determine the elements of R. (5 marks)

d) Let $f: \mathbb{N} \to \mathbb{N}$ be the function defined by $f(x) = 2x^2 + 3$. Show that f is neither surjective nor injective. (5 marks)

e) Majority of Kenyan private vehicle number plates have the format $K \ast\ast\ast$ where each character $\ast$ is alphabetic letter and each character is a numeric digit (e.g. KAB541X). Determine the number of distinct number plates having this format that can be produced.

(5 marks)

f) Determine the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 = 91$. (4 marks)

QUESTION TWO (20 MARKS)

a) Explain the meaning of the terms

i. an *anti-symmetric* binary relation on a set X, (2 marks)

ii. an r-*combination with repetition* on a finite set X. (2 marks)

b) Let $\mathbb{R}^+$ denote the set of all positive real numbers and consider the relation from $\mathbb{R}^2$ to $\mathbb{R}$ whose elements are given by $S = \{(x, \pm\sqrt{x}: x \in \mathbb{R}^+)\}$. Determine whether S is a function. (3 marks)

c) Use the Euclidean algorithm

i. to find $d = gcd(3385,641)$, (4 marks)

ii. express $gcd(3385,641)$ in the form $d = 3385m + 641n$. (5 marks)

d) In a particular general election, there are eight candidates vying for a given position. What is the minimum number of valid votes cast so as to ensure that at least one candidate receives not less than 2,345,611 votes? Justify your answer. (4 marks)

QUESTION THREE (20 MARKS)

a) Consider the set $A = \{2,6,7,21\}$ and let R be a binary relation on A whose elements are

$$R = \{(2,6), (6,7), (2,7), (7,2)\}$$

Determine if R is a transitive binary relation. (3 marks)

b) Give that $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$, explain why there are no injective functions $f: A \to B$. (3 marks)

c) Let a, b, m be integers with $m > 0$ and suppose that $a \equiv b(mod\ m)$.

i. Prove that $a + x \equiv b + x(mod\ m)$ for any integer x. (4 marks)

ii. Solve the equation $374x \equiv 51(mod\ 289)$ (5 marks)

d) Each user on a computer system has a password which is six to eight characters long. Each character in a generated password can be a lowercase alphabetic letter or a numeric digit. Each password must contain at least one digit. Determine the possible number of unique passwords that can be generated. (5 marks)

QUESTION FOUR (20 MARKS)

a) Explain the meaning of the following terms

i. a *binary relation* from a set X to a set Y, (2 marks)

ii. an r-*permutation* on a finite set S with n elements. (2 marks)

b) Suppose that $f: X \to Y$ and $g: Y \to Z$ are injective functions. Prove that the composition function $g \circ f: X \to Z$ is an injective function as well. (4 marks)

c) Students enrolled in the school of sciences are supposed to do a research project towards the end of their studies. Currently, the school has proposed and published 53 projects in mathematics, 35 projects in chemistry and 57 projects in physics. Each student is expected to choose 2 projects in mathematics, 2 projects in chemistry and 2 projects in physics before submitting the choices to the school for further considerations. Determine the number of choices that each student has in fulfilling the required expectation. (4 marks)

d) Consider the sequence $\{a_n\}$ defined by $a_n = 3a_{n-1} - 2a_{n-2}$. Solve this recurrence relation subject to the initial condition $a_0 = 1, a_1 = 3$. (4 marks)

e) Suppose that n, k are positive integers such that $1 \leq k \leq n$. Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$. (4 marks)

QUESTION FIVE (20 MARKS)

a) Let $\sim$ be a binary relation defined on $\mathbb{R}^2$ by $(a, b) \sim (x, y)$ if and only if $(a, b) = \lambda(x, y)$ for some real number $\lambda \neq 0$.

i. Prove that $\sim$ is an equivalence relation on $\mathbb{R}^2$. (5 marks)

ii. Describe the equivalence class of $(1,1)$ under the equivalence relation $\sim$ defined above. (2 marks)

b) Prove that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$. (5 marks)

c) Assume that a country currently with a population of 45 million people has a population growth of 3.6%.

(i) Determine how big its population will be in 15 years time. (4 marks)

(ii) If the country receives 230,000 new immigrants every year, determine its population in 15 years time assuming that all the immigrants arrive at the end of the year and reproduce at the same rate as the native population. (4 marks)