



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS & PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 261: PROBABILITY AND STATISTICS II

DATE: 25/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Determine the $E(x_1 + x_2)$ from the Bivariate table below. (3 marks)

		x_2	
		0	1
x_1	0	$\frac{1}{6}$	0
	1	$\frac{1}{3}$	$\frac{1}{3}$
	2	0	$\frac{1}{6}$

- b) If the joint probability density function of X and Y is given by:

$$f(x, y) = 3y, 0 \leq y \leq 1, 0 < x < 1. \text{ Determine } F\left(\frac{1}{3}, \frac{1}{2}\right) \quad (6 \text{ marks})$$

- c) The joint probability density function for x and y is given by $f(x, y) = \frac{1}{21}(x + y)$

$$x = 1, 2, 3.$$

$$y = 1, 2$$

- i. Determine marginal distribution of x and y (6 marks)
- ii. Prove for independence (4 marks)
- d) Show that $E(X+Y)=E(X)+E(Y)$ if X and Y are independent continuous random variables (5 marks)
- e) Given that $f(x, y) = \begin{cases} 2x, & 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$
Determine $p(0 < x < \frac{1}{3}, \frac{1}{2} < y < 1)$ (6 marks)

QUESTION TWO (20 MARKS)

- a) Given y_1 and y_2 have the joint probability density function (pdf) given by

$$f(y_1, y_2) = \begin{cases} K(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$
 - Determine the value of the constant K . (4 marks)
 - Determine $p(y_1 \leq \frac{3}{4}, y_2 \geq \frac{1}{2})$ (4 marks)
- b) If x and y have joint probability density function $f(x, y) = \frac{8x^2}{7y^3}$ for $1 \leq x, y \leq 2$
 - i. Determine the marginal distribution of x and y (8 marks)
 - ii. Determine the conditional density function of y given $X = x$ (2 marks)
 - iii. Determine whether x and y are independent. (2 marks)

QUESTION THREE (20 MARKS)

- a) Suppose X is a continuous and Y is discrete random variables with a joint p.d.f given by:

$$f(x, y) = \frac{yx^{y-1}}{3}, 0 < x < 1, y \in 1, 2, 3.$$
 Determine
 - i. $f_y(y)$ (5 marks)
 - ii. $E(X)$ (5 marks)

b) Consider the Bivariate table below

		y	
		0	1
x	0	1/8	0
	1	2/8	1/8
	2	1/8	2/8
	3	0	1/8

- Determine $E(x)$ (2 marks)
- Determine the covariance of x and y (4 marks)
- The marginal probability mass function of y (4 marks)

QUESTION FOUR (20 MARKS)

- Let $y_1 \sim B(n_1, p)$ and $y_2 \sim B(n_2, p)$. Determine the moment generating function of $y_1 + y_2$ given that y_1 and y_2 are independent binomial random variables. (6 marks)
- The joint pdf of x and y is given by that $f(x, y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

- Determine:
- μ_x and μ_y (4 marks)
 - d_x^2 and d_y^2 (4 marks)
 - The correlation coefficient between x and y (6 marks)

QUESTION FIVE (20 MARKS)

- Let y be a continuous random variable with a pdf defined by that

$$f(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Using the change of variable technique determine the density of $x = 8y^3$. (8 marks)

- Given that $E(y/x) = \alpha + \beta x$, where E denotes expectation, x and y denotes random variables and α and β are constants. Show that:

$$\beta = \frac{E(xy) - E(x)E(y)}{E(x^2) - (E(x))^2} = \frac{\text{cov}(x, y)}{\text{var}(x)} \quad (12 \text{ marks})$$