MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SUPPLEMENTARY EXAMINATION

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: 31/8/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Attempt **Question one (compulsory)** and any ***other two questions***.

1. a) Define the following terms as used in differential equations context ;
   - i.) Analytic function (2 marks)
   - ii.) Power series solution (2 marks)
   - iii.) Ordinary point of a differential equation (2 marks)
   - iv.) Regular singular point of a differential equation (2 marks)

   b) using the power series method solve the differential equation $y' - xy = 0$ composing your result to those obtained analytically. (5 marks)

   c) determine the polynomial solution to the Legendre equation

   $(1 - x^2)y'' - 2xy' + 12y = 0$ (5 marks)

   d) using the Frobenius method solve the differential equation

   $4xy'' - 2y' + y = 0$ (5 marks)

   e) determine the Laplace transform of $f(t) = e^{at}$ where a is a constant.

   (3 marks)

f) determine the half range sine Fourier series representing the function $f(x)$ defined by;

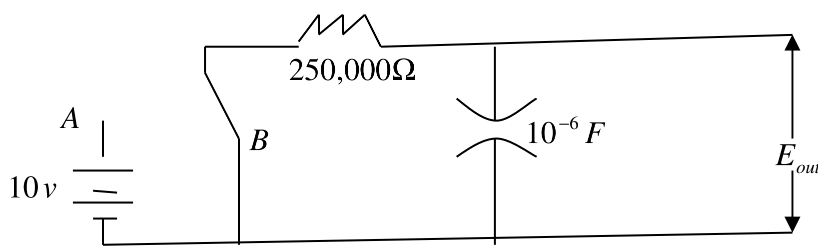
$$f(x) = 1 + x \quad ; \quad 0 < x < \pi$$

$$f(x) = f(x + 2\pi)$$ (4 marks)

2. a.) use Laplace transforms method to solve the second order differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}$ given that at $t = 0$; $x = 5$ and $\frac{dx}{dt} = 7$ (7 marks)

b.) determine the inverse of $\frac{5s+1}{(s-4)(s+3)}$ (3 marks)

c.) suppose the capacitor in the circuit whose diagram is given below initially has zero charge and that there is no initial current. At time $t = 2\sec onds$, the switch is thrown from position B to A, held there for $1 \sec ond$, then switched back to B. calculate the output voltage E_{out} on the capacitor. (10 marks)



3. a.) use the Bessel's function identities to express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$

(3 marks)

b.) Evaluate $\int x^3 J_0 dx$ (3 marks)

c.) determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x$ and $\cos x$

(4 marks)

d.) show that $xF(1,1,2;-x) = \ln(1+x)$ (4 marks)

e.) determine the general solution to Bessel's equation of order zero. (6 marks)

4. a.) determine the Fourier's series expansion for;

$$f(x)=\begin{cases}0 & ; -\pi \le x < \dfrac{-\pi}{2} \\ \cos x & ; \dfrac{-\pi}{2} \le x < \dfrac{\pi}{2} \\ 0 & ; \dfrac{\pi}{2} \le x < \pi\end{cases}$$ (4 marks)

b.) determine a Fourier cosine series for $f(x)=e^{x}$ on $(0,\pi)$ (4 marks)

c.) i.) define a Dirac delta function (1 mark)

ii.) state the convolution theorem of Fourier transforms. (3 marks)

iii.) given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t)*g(t)=\int_{-\infty}^{\infty} f(x)g(t-x)dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t)=u(t)$ and $g(t)=\begin{cases}\sec^2 t & |t|<\dfrac{\pi}{4} \\ 0 & \text{otherwise}\end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t)=\dfrac{1+\tan^2 t}{1+\tan t}$ (4 marks)

d.) find the inverse transform of $f(\omega)=\dfrac{1}{2\pi(a+j\omega)^2}$ where $a>0$ (5 marks)

5. a.) Solve $$\frac{\partial^2 u}{\partial x \partial y} = \sin x \sin y$$ subject to the conditions;

$$\frac{\partial u}{\partial x}\left(x, \frac{\pi}{2}\right) = 2x$$

$$u\ (\pi, y) = 2\sin y$$

(5 marks)

b.) use the method of separation of variables to find the solution of the equation;

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$ which satisfies the conditions;

i.) $u(0,t) = u(a,t) = 0 \qquad \forall\ t \geq 0$

ii.) $u(x,0) = x \qquad for\ 0 \leq x \leq a$

(15 marks)