MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 106: ENGINEERING MATHEMATICS III

DATE: 16/6/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer Question one (compulsory) any other two Questions

QUESTION ONE (30 MARKS)

a) Define a scalar and a vector quantity and give two examples to each (2 marks)

b) Solve for x, y and z given the following matrix equation.

$$\begin{bmatrix} x & 2 & -3 \\ 5 & y & 2 \\ 1 & -1 & z \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{bmatrix}$$ (3 marks)

c) If $a = 2i + 3j + 4k$, $b = 4i - j - k$. Evaluate

i. $|a+b|$ (2 marks)

ii. The angle between a and b (3 marks)

d) Given the vectors $a = 3i + 3j - 2k$, $b = -i + 3j + 4k$ and $c = 4i - 2j - 6k$.

Find a unit vector coplanar with a and b but perpendicular to c (5 marks)

e) Solve by Cramer's rule, the following system of equations

$x + y + z = 7$

$2x + 3y + 2z = 17$ (5 marks)

$4x + 9y + z = 37$

f) Find the eigen values of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$ (5 marks)

g) Find an equation of the plane passing through the points $P = (-1,2,1)$, $Q = (0,-3,2)$, and $R(1,1,-4)$ (5 marks)

QUESTION TWO (20 MARKS)

a) The position vectors P, Q, R are respectively $P = i + j - k$, $Q = 2k$ and $R = j - 2k$. Find

$$\frac{|QP|}{|PR|}$$ (2 marks)

b) Find the area of a triangle whose vertices have the position vectors $a = i - j + 2k$, $b = 2j + k$ and $c = j + 3k$ (4 marks)

c) If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 3 \\ 4 & 1 & -1 \end{bmatrix}$; $C = \begin{bmatrix} 7 & 2 \\ 1 & 0 \\ -2 & 0 \end{bmatrix}$ verify that $(AB)C = A(BC)$

(5 marks)

d) Solve the linear system using matrix method

$3x - y + 2z = 13$

$2x + y - z = 3$ (9 marks)

$x + 3y - 5z = -8$

QUESTION THREE (20 MARKS)

a) If $a = 2i - j + k$, $b = 3i + j - k$, $c = i + \lambda j$, find the value of λ for which $a \cdot (b \times c) = -25$ (3 marks)

b) If $2A + B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & 0 \end{bmatrix}$ and $3A + 2B = \begin{bmatrix} 4 & 6 & 1 \\ 2 & 3 & 5 \end{bmatrix}$, find A and B (3 marks)

c) Find the equation of a line passing through the points $A = (1, 0, -1)$ and $B = (-1, -1, 0)$

(4 marks)

d) Find a unit vector which should lie on the plane determined by $a = 2i + j + k$, $b = i + 2j + k$ and perpendicular to $c = i + j + 2k$ (4 marks)

e) Solve the equations using Gauss's elimination method

$$x + y + z = 6$$
$$3x + 3y + 4z = 20$$
$$2x + y + 3z = 13$$

(6 marks)

QUESTION FOUR (20 MARKS)

a) If $a = i + j + k$; $b = 2i - 3j + 2k$; $a = 2i - j + k$, $c = i - 2j - k$ find $(a \times b) \times c$

(3 marks)

b) Find parametric and symmetric equations of the line passing through the points $(1, -1, 2)$ and $(2, 1, 5)$ (4 marks)

c) Find an equation of the plane passing through the point $P = (1, 6, 4)$ and containing the line defined by $R(t) = (1 + 2t, 2 - 3t, 3 - t)$ (5 marks)

d) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(8 marks)

QUESTION FIVE (20 MARKS)

a) If $a = 2i + j + k$ and $b = i + j - 2k$, find the projection of a in the direction of b

(3 marks)

b) Given that matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, find

i. The eigen values and the corresponding eigen vectors (8 marks)

ii. Obtain $M = [x_1, x_2, x_3]$, hence M^{-1} (the inverse of M) (6 marks)

iii. Obtain AM and finally $M^{-1}AM$ (3 marks)