MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 202: ENGINEERING MATHEMATICS VII

DATE: 14/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS TO CANDIDATES

Answer **Question One** and **Any Two** Other Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Form the differential equation associated with $y^2 = 4ax$ (3 marks)

b) Find the differential equation of the family $y = Ae^{2x} + Be^{-2x}$ (5 marks)

c) Find the general solution of $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$ (5 marks)

d) Apply the method of separation of variables to solve $\frac{dy}{dx} + xy = xy^3$ (7 marks)

e) A chain coiled up near the edge of a smooth table begins to fall over the edge. When a length x of the chain has fallen, the equation of motion is given by $\frac{d}{dt}(mxv) = mxg,$ when m is the mass of the chain per unit length, v is the speed, g is the acceleration due to gravity and t is the time. Show that the speed is given by

$$v = \sqrt{\left(\frac{2}{3}\right)gx}$$

(10 marks)

QUESTION TWO (20 MARKS)

a) Show that

$Ax^2 + By^2 = 1$ is the solution of

$$x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - y\frac{dy}{dx} = 0$$ (3 marks)

b) Use the method of undetermined coefficient to solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 111e^{2x}\cos 3x$$ (7 marks)

c) Find the orthogonal trajectories of the family

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$$ (10 marks)

QUESTION THREE (20 MARKS)

a) Solve $D^2 + 2D + 1 = x\cos x$ (6 marks)

b) Obtain the equation of the curve satisfying this equation and passing through the origin $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ (6 marks)

c) A cyclist, moving on a level road at $4m/s$ stops pedaling and free wheels to rest. The retardation of the cycle has two components a constant $0.08m/s^2$ due to friction in the working parts and resistance of $0.02v^2/s^2$ where v is the speed in meters per second. What distance traversed by the cyclist before it comes to rest? (8 marks)

QUESTION FOUR (20 MARKS)

a) Find the general solution to $(D^4 - 81)y = 0$ (4 marks)

b) Find the general solution of

$y = x + 2\tan^{-1} p$ (6 marks)

c) Reduce the given differential equation

$x^2\left(\frac{dy}{dx}\right)^2 + y(2x+y)\frac{dy}{dx} + y^2 = 0$ to Clairut's form by using the substitution $y = u,\ xy = v,$ and find its singular solution. (10 marks)

QUESTION FIVE (20 MARKS)

a) Solve

$$(2x+3y-5)\frac{dy}{dx}+(3x+2y-5)=0$$ (10 marks)

b) If the population of a country doubles in 50 years. In how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? (10 marks)