MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 360: MULTIVARIATE METHODS 1

DATE: 2/8/2017 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Define the following terms:

i. Random vector (2 marks)

ii. Mean vector, $\bar{x}$ (2 marks)

iii. Correlation matrix (2 marks)

b) Consider the following data with three variables X, Y and Z.

X	Y	Z
3	9	12
4	2	8
0	6	6
2	5	4
1	8	10

Calculate the:

i. Mean vector (3 marks)

ii. Covariance matrix (5 marks)

iii. Correlation matrix (3 marks)

c) Given the covariance matrix $\Sigma=\begin{bmatrix}3 & \frac{1}{3}\\ \frac{1}{3} & 2\end{bmatrix}$. Compute $\operatorname{var}(3x+4y-5)$. (3 marks)

d) Given $\underline{X}'=(X_1,X_2,X_3,X_4,X_5)$, $\underline{\mu}'=(2,4,-1,3,0)$, $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & -2\end{pmatrix}$. Partition

$$\underline{X}_1=\begin{pmatrix}X_1\\ X_2\end{pmatrix} \text{ and } \underline{X}_2=\begin{pmatrix}X_3\\ X_4\\ X_5\end{pmatrix},\ \Sigma=\begin{bmatrix}4 & -1 & \frac{1}{2} & -\frac{1}{2} & 0\\ -1 & 3 & 1 & -1 & 0\\ \frac{1}{2} & 1 & 6 & 1 & -1\\ -\frac{1}{2} & -1 & 1 & 4 & 0\\ 0 & 0 & -1 & 0 & 2\end{bmatrix}.$$

Find $E(A\underline{X}_2)$ and $\operatorname{var}(A\underline{X}_2)$ (6 marks)

e) Explain two properties of covariance matrix for a multivariate distribution. (4 marks)

QUESTION TWO (20 MARKS)

a) Let $\underline{X}\sim N_P(\underline{u},\Sigma)$, $\Sigma>0$. Use the characteristic function technique to show that if $\underline{Y}=A\underline{X}$ where A is a non-zero of constants, then $\underline{Y}\sim N_P(A\underline{u},A\Sigma A')$.
(5 marks)

b) Let $\underline{X}$ be a three variate random vector such that:

$$\operatorname{var}(\underline{X})=\begin{bmatrix}5 & 2 & 3\\ 2 & 3 & 0\\ 3 & 0 & 2\end{bmatrix}=\Sigma$$

Determine the variance-covariance matrix of the random vector $\underline{Y}=(Y_1,Y_2,Y_3)'$ where

$Y_1=X_1+X_2$
$Y_2=X_1+X_3$ (6 marks)
$Y_3=X_2+X_3$

c) Let $\underline{X}$ be a random vector of p-dimension.

i. Write down the multivariate normal distribution, $f(\underline{X})$ of the random vector $\underline{X}$.
(2 marks)

ii. Consider the bivariate normal distribution with parameters $\mu_1,\mu_2,\delta_1,\delta_2$ and ρ_{12}. write down the bivariate normal distribution in terms of the above parameters. (5 marks)

iii. What happens if $\delta_{12} = 0$, $\mu = 0$ (2 marks)

QUESTION THREE (20 MARKS)

a) Define the term characteristic function of a random variable X. (2 marks)

b) State any three properties of the characteristic function. (3 marks)

c) Suppose $\underline{X} \sim N_P(0, 1)$. Show that the characteristic function of X is given by:
$\phi_{\underline{x}}(t) = \exp(-\frac{t^2}{2})$. (8 marks)

d) Suppose $\underline{X} \sim N_P(\underline{u}, \Sigma)$, $\Sigma > 0$ and $\underline{y} = \Sigma^{-\frac{1}{2}}(\underline{X} - \underline{\mu}) \sim N(\underline{0}, I_P)$. Use the characteristic function technique to show that $\phi_{\underline{x}}(t) = \exp(i\underline{t}'\underline{\mu} - \frac{1}{2}\underline{t}'\Sigma\underline{t})$. (7 marks)

QUESTION FOUR (20 MARKS)

a) Suppose $\underline{X}' = (X_1, X_2, X_3) \approx N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu}' = (20, 14, 22)$ and the dispersion matrix defined by $\Sigma = \begin{bmatrix} 2 & 6 & 4 \\ 6 & 8 & 2 \\ 4 & 2 & 9 \end{bmatrix}$. .Partition $\underline{X}_1 = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ and $\underline{X}_2 = (X_3)$,

Find

i. $E(\underline{X}_1) = \underline{\mu}_1$, $E(\underline{X}_2) = \underline{\mu}_2$

ii. $Cov(\underline{X}_1, \underline{X}_2) = \Sigma_{12}$, $Cov(\underline{X}_2, \underline{X}_1) = \Sigma_{21}$

iii. $\text{var}(\underline{X}_1) = \Sigma_{11}$, $\text{var}(\underline{X}_2) = \Sigma_{22}$ (6 marks)

b) Determine $f(\underline{X}_1/\underline{X}_2)$ *and* $f(\underline{X}_2/\underline{X}_1)$ given that :

$$E(\underline{X}_1/\underline{X}_2) = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\underline{X}_2 - \underline{\mu_2})$$

$$E(\underline{X}_2/\underline{X}_1) = \mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(\underline{X}_1 - \underline{\mu}_1)$$

$$\text{var}(\underline{X}_1/\underline{X}_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

$$\text{var}(\underline{X}_2/\underline{X}_1) = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$$ (14 marks)

QUESTION FIVE (20 MARKS)

a) Let $\underline{X}_1 = \begin{pmatrix} X_1 \\ X_2 \\ ' \\ ' \\ X_r \end{pmatrix}$ and $\underline{X}_2 = \begin{pmatrix} X_{r+1} \\ X_{r+2} \\ ' \\ ' \\ X_p \end{pmatrix}$ be partitioned matrices of $\underline{X}$. Let the mean and the corresponding dispersion matrix defined by:

$$\underline{\mu} = \begin{pmatrix} \underline{\mu}_1 \\ \mu_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

Show that the marginal distribution of $\underline{X}_1 \approx N_r(\underline{\mu}_1, \Sigma_{11})$ (7 marks)

b) Suppose $\underline{y}$ is $N_r \approx (\underline{\mu}, \Sigma)$ where

$$\underline{\mu} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 4 \end{pmatrix}$$

i. Find the distribution of $z = 3y_1 - 2y_2 + 4y_3$ (7 marks)

ii. Determine the marginal distribution of y_2. (6 marks)