MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

DIPLOMA IN ELECTRICAL AND ELECTRONICS ENGINEERING

DIPLOMA IN MECHANICAL ENGINEERING

DIPLOMA IN CIVIL ENGINEERING

ECU 0202 ENGINEERING MATHS VI

DATE: 20/01/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer question <u>one</u> and any other <u>two</u>

QUESTION ONE (30 MARKS)

a) Define the following terms in numerical methods

i. Extrapolation (2 marks)

ii. iteration (2 marks)

b) Derive the Newton -Raphsons interpolation formula (5 marks)

c) Use the Newton Raphsons method to show that if x_0 is an approximation to the root of the equation $x^3 + 2x^2 - 5x = 1$ then the better approximation is given by

$$x_{n+1} = \frac{2x^3+2x^2+1}{3x^2+4x-5}$$ (5 marks)

d) Given $x_n = 1.2345$ $x_{n+1} = 1.2447$ $x_{n+2} = 1.3124$

$x_{n+3} = 1.3233$ $f(x_n) = 12.5674$ $and\ f(x_{n+2}) = 13.9831$

Use linear interpolation and extrapolation to calculate

$f(x_{n+1})$ $and\ f(x_{n+2})$ to 5 decimal places (5 marks)

e) i Obtain the cubic polynomial in x which takes on the values $-3, 3, 11, 27, 57\ and\ 107$

when the values of $x = 0, 1, 2, 3, 4 \text{ and } 5$ (6 marks)

ii Hence take $x_0 = 7$ to find the root of the equation correct to four decimal places.

(5 marks)

QUESTION TWO (20 MARKS)

a) A function $f(x)$ is defined by the data in the table below

X	0	1	2	3	4	5	6
F(x)	-1	2	11	38	95	194	347

Use the Newton Gregory forward difference interpolation formulae to estimate

i. $f(4.8)$ (5 marks)

ii. $f(6.4)$ (5 marks)

b) Given that x_n is an approximation to the root of the equation

$x^3 - 6x^2 - 6x - 8 = 0$

i) show using the Newton-Raphson method that a better approximation to the root is given by

$x_{n+1} = \frac{2x^3 - 6x^2 + 8}{3x^2 - 12x + 6}$

ii) Hence by taking $x_1 = 5$ find the root of the equation correct to five decimal places. (10 marks)

QUESTION THREE (20 MARKS)

a) The values in the table below were obtained from an experiment

X	-0,5	0.0	0.5	1.0	1.5
F(x)	1.327	1.382	1.416	1.452	1.513

Use the newton Gregory forward difference interpolation formula to find the polynomial f(X)which corresponds to the values given above.

Hence evaluate

i) $f(-0.52)$

ii) ii) $f(1.2)$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Given that $x_1 = 4$ as an approximation to $\sqrt{17}$ use the iteration method to find a better approximation to the root. (5 marks)

b) The table below satisfies a function $f(x)$

X	0.1	0.3	0.5	0.7	0.9	1.1	1.3
F(x)	0.003	0.067	0.148	0.248	0.379	0.518	0.697

Use the Newton-Gregory finite difference interpolation formulae to determine the value of $f(-0.5)$ (10 marks)

QUESTION FIVE (20 MARKS)

a) The table below satisfies a function $f(x)$

X	-1	0	1	2	3	4	5
F(x)	1	1	9	43	121	261	481

Evaluate

i. f(2.5) (5 marks)

ii. f(4.8) (5 marks)

b) i Given the function $x^4 + 5x - 20 = 0$ show that a better approximation to the root is given by

$$x_{n+1} = \frac{3x_n^4+20}{4x_n^3+5}$$ (5 marks)

ii Taking $x_0 = 1.9$ calculate the root of the equation correct to five decimal places. (5 marks)