MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 301: ENGINEERING MATHEMATICS X

DATE: 14/4/2023 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS TO CANDIDATES.

Answer question one (**compulsory**) and any other **two** questions

QUESTION ONE (30 MARKS)

a) Find the real roots of the equation $xe^x - 2$ correct to two decimal places using Newton Raphson method. (4 marks)

b) Find the approximate value of $y = \int_0^{\pi} sin\, x \;\; dx$ using trapezoidal rule with six intervals (4 marks)

c) Find the cubic polynomial which takes the following values $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$ hence or otherwise obtain the value of $y(8)$. $(5\ marks)$

d) Construct a forward difference table for the following values of x and y $(5\ marks)$

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

e) Evaluate $f(15)$ given the following table values $(5\ marks)$

x	10	20	30	40	50
f(x)	46	66	81	93	101

f) Given the values of $x(in\ degrees)$ as 15^0, 20^0 ,25^0,30^0,35^0,40^0 and that $y(x) = sin\, x$. Determine the value of $sin\ 38°$. $(7\ marks)$

QUESTION TWO (20 MARKS)

a) Using Romberg's Integration method, find the value of $\int_1^{1.8} y(x)\ \ dx$ starting with Trapezoidal rule for the tabular values. $(10\ marks)$

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
y=f(x)	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.828	3.107

b) Find Lagrange's interpolation polynomial fitting the points $y(1) = -3,\ y(3) = 0,$ $y(4) = 30\ \ y(6) = 132$ hence find y(5). $(10\ marks)$

QUESTION THREE (20 MARKS)

a) Determine the value of y when $x = 0.1 given\ \ that\ \ y(0) = 1\ and\ \ that$ $y' = x^2 + y\ \ taking\ h = 0.05$ using Modified Euler Method.
Given the differential equation $y'' - xy' - y = 03q2w$
$with\ the\ initial\ conditions\ y(0) = 1.$ $(10\ marks)$

b) Evaluate $\sqrt{155}$ by using Lagrange's interpolation formula from the following data $(5\ marks)$

x	150	152	154	156
$y = \sqrt{x}$	12.247	12.239	12.41	12.49

c) Use Newton-Raphson method to find a real root correct to 3 decimal points up to the fourth iteration of the equation $sin\ x = \frac{x}{2}$ given that the root lie between $\frac{\pi}{2}$ and π. $(5\ marks)$

QUESTION FOUR (20 MARKS)

a) Given $\frac{dy}{dt} = \frac{y-t}{y+t}$ with initial condition $y = 1\ at\ t = 0$ find y approximately at x=0.1, in five steps using Euler Method. $(8\ marks)$

b) Use fourth-order Runge -Kutta method to solve $\frac{dy}{dt} = t + y$ with initial condition $y(0) = 1$ from $t = 0\ to\ t = 0.4\ taking\ h = 0.1.$ $(12\ marks)$

QUESTION FIVE (20 MARKS)

a) Find the numerical solution of the initial value problem described below using Second order Runge-Kutta method $(8\ marks)$

$\frac{dy}{dt} = \frac{y-t}{y+t}\quad y(0) = 1\ \ at\ \ x = 0.4\ and\ \ taking\ h = 0.2$

b) Using Taylor's series method find the solution of the initial value problem

$$\frac{dy}{dt} = t + y, \qquad y(1) = 0$$

At t=1.2 with h=0.1 and compare the results with the closed form solution. $(12\ marks)$