MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 305: COMPLEX ANALYSIS I

DATE: 20/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 MARKS (COMPULSORY)

a) Find the fixed points of the bilinear transformation $w = \frac{3z-4}{z-1}$ (3 marks)

b) If $w = f(z) = 3z + 4 - 5i$, find the value of w which corresponds to $z = -3 + i$ (3 marks)

c) Find the image of the point $(4,3)$ on $z-$plane under the transformation $w = 2z^2 + 3$ (4 marks)

d) Evaluate $\oint \frac{dz}{z-a}$ over a closed curve C and **a** is inside C (5 marks)

e) Simplify $\frac{(\cos 4\theta + i\sin 4\theta)(\cos 3\phi + i\sin 3\phi)}{(\cos\theta + i\sin\theta)(\cos 2\phi + i\sin 2\phi)}$ (5 marks)

f) Find the value of the derivative of $f = \frac{z-i}{z+i}$ at $z = i$ (5 marks)

g) Find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)}$ at all its poles (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the function $f(z) = \frac{z - \sin z}{z^2}$

i. State the singularity of $f(z)$ (2 marks)

ii. What kind of the singularity of $f(z)$ (2 marks)

iii. Find the Laurent series of $f(z)$ (5 marks)

iv. What is the region of convergence of $f(z)$ (1 mark)

b) Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane. Hence find harmonic conjugate function v of u. (10 marks)

QUESTION THREE (20 MARKS)

a) Using definition of e^z prove that $e^{z_1}.e^{z_2} = e^{z_1+z_2}$ (5 marks)

b) Expand $f(z) = \frac{1}{(z-1)(z-3)}$ about $z = 1$ in Laurent series (5 marks)

c) Use residue theorem to show that $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta} = \frac{2\pi}{3}$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Determine whether $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ is continuous at $z = i$. Hence determine its limit at $z = i$ (5 marks)

b) Prove that $\sin z = \sin x \cosh y - i \cos x \sinh y$ (5 marks)

c) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively. (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the poles of $\frac{e^{iz}}{z^2(z^2+2z+2)}$ (5 marks)

b) Show that $f(z) = e^z$ is a periodic function with period $2k\pi i$ (5 marks)

c) Expand $f(z) = \dfrac{3}{z^2(z-3)^2}$ in a Laurent series at $z = 3$ (5 marks)

d) Evaluate $\int \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$ (5 marks)