



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF
EDUCATION AND BACHELOR OF ARTS

SMA 305: COMPLEX ANALYSIS I

DATE:..... TIME:.....

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

a.) Find the fixed points of the bilinear transformation $w = \frac{3z - 4}{z - 1}$ (3 marks)

b.) If $w = f(z) = 3z + 4 - 5i$, find the value of w which corresponds to

$$z = -3 + i \quad (3 \text{ marks})$$

c.) Find the image of the point $(4,3)$ on z -plane under the transformation $w = 2z^2 + 3$ (4 marks)

d.) Evaluate $\oint \frac{dz}{z-a}$ over a closed curve C and a is inside C (5 marks)

e.) Simplify $\frac{(\cos 4\theta + i \sin 4\theta)(\cos 3\phi + i \sin 3\phi)}{(\cos \theta + i \sin \theta)(\cos 2\phi + i \sin 2\phi)}$ (5 marks)

f.) Find the value of the derivative of $f = \frac{z-i}{z+i}$ at $z = i$ (5 marks)

g.) Find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)}$ at all its poles (5 marks)

QUESTION TWO (20 MARKS)

a.) Consider the function $f(z) = \frac{z - \sin z}{z^2}$

- i. State the singularity of $f(z)$ (2 marks)
- ii. What kind of the singularity of $f(z)$ (2 marks)
- iii. Find the Laurent series of $f(z)$ (5 marks)
- iv. What is the region of convergence of $f(z)$ (1 marks)

b) Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane. Hence find harmonic conjugate function v of u . (10 marks)

QUESTION THREE (20 MARKS)

a.) Using definition of e^z prove that $e^{z_1} \cdot e^{z_2} = e^{z_1+z_2}$ (5 marks)

b.) Expand $f(z) = \frac{1}{(z-1)(z-3)}$ about $z = 1$ in Laurent series (5 marks)

c.) Use residue theorem to show that $\int_0^{2\pi} \frac{d\theta}{5+4 \sin \theta} = \frac{2\pi}{3}$ (10 marks)

QUESTION FOUR (20 MARKS)

a.) Determine whether $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z-i}$ is continuous at $z = i$. Hence

determine its limit at $z = i$ (5 marks)

b.) Prove that $\sin z = \sin x \cosh y - i \cos x \sinh y$ (5 marks)

c.) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively. (10 marks)

QUESTION FIVE (20 MARKS)

a.) Determine the poles of $\frac{e^{iz}}{z^2(z^2+2z+2)}$ (5 marks)

b.) Show that $f(z) = e^z$ is a periodic function with period $2k\pi i$ (5 marks)

c.) Expand $f(z) = \frac{3}{z^2(z-3)^2}$ in a Laurent series at $z = 3$ (5 marks)

d.) Evaluate $\int \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$ (5 marks)