



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

SMA 262/SAC 300 DIFFERENTIAL EQUATIONS

DATE: 18/3/2022

TIME: 2.00-4.00 PM

INSTRUCTIONS:

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ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) Distinguish between the following terms
- i. An ordinary differential equation and partial differential equations (2 marks)
 - ii. Order and degree of a differential equation. (2 marks)
- b) Solve the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$ which defines a family of circles. (3 marks)
- c) Find the general solution of the Lagrange equation $2p + 3q = 1$ (4 marks)
- d) Solve $(x^3 + y^3)dx - 3xy^2dy = 0$ (3 marks)
- e) Solve the Bernoulli's differential equation $\frac{dy}{dx} + y = xy^2$. (5 marks)
- f) Solve the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 29y = 0$, $y(0) = 0, y'(0) = -3$ using power series method. (4 marks)
- g) Solve using a suitable integrating factor $-xdx + (y + x^4)dy = 0$ (4 marks)
- h) Find the general solution of the equation $(1 + x^2)\frac{dy}{dx} - xy = x + x^3$ (3 marks)

QUESTION TWO (20 MARKS)

- a) Solve the ordinary differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = e^{2x} + 3x^2 + 4x - 5 \quad (7 \text{ marks})$$

- b) Classify and solve the ordinary differential equation

i. $xdx - y^2dy = 0$ (3 marks)

ii. $\frac{dy}{dx} = \frac{x^2}{y^2}$ (2 marks)

iii. $\frac{dy}{dx} = -\frac{x}{x+y}$ (3 marks)

iv. $(xy + x^2)dx - dy = 0$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Solve the following differential equations by first reducing the order of the equation

i. $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = 0$ (5 marks)

ii. $\frac{d^2y}{dx^2} + y = 0$ (5 marks)

- b) Solve the following;

i. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$ (4 marks)

ii. $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0, \quad y(0) = 2, y'(0) = -3$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Show that $y = C_1e^x + C_2e^{-x} + C_3e^{2x}$ is a solution of the differential equation

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0 \quad (5 \text{ marks})$$

- b) Solve the ordinary differential equations

i. $\frac{dy}{dx} = \frac{xy^2+x}{yx^2+y}$ (5 marks)

ii. $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$ (5 marks)

iii. $(x - 2y - 1)dx + (4x - 3y - 6)dy = 0$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Solve $y''' + 5y'' - 24y' - 36y = 0$ (5 marks)
- b) Find the primitive solution to the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x - 3$. Find the particular solution if the curve passes through (0,0) and (1,0). (5 marks)
- c) Determine the Wroskian of the functions e^x , e^{2x} and e^{3x} and show that they are linearly independent (5 marks)
- d) Solve $y'' + 4y = 2 - 4x^2 + e^x$ (5 marks)