



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF ECONOMICS

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 104: CALCULUS I

DATE: 30/4/2019

TIME: 11:00 – 1:00 PM

INSTRUCTIONS:

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Consider the function $f(x) = \ln\left(\frac{\sin 3x}{x}\right)$. Express the function as the composite of two functions (5 marks)
- b) Evaluate $\lim_{x \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$ (5 marks)

- c) Prove that the function $f(x) = \cos x$ is continuous for every value of x on R (5 marks)
- d) Approximate $\sin 29^\circ$ using small changes given that $1^\circ = 0.0175 \text{ radians}$ (5 marks)
- e) Use the first principle to evaluate $\frac{dy}{dx}$ for $y = x^2 + 3$ (5 marks)
- f) A particle moves in the plane according to the law $x = t^2 + 2t$, $y = 2t^3 - 6t$. Determine the slope of the normal line when $t = 0$. (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = 3x^5 - 20x^3$

- a) Determine using the second derivative
- Local maximum point (4 marks)
 - Local minimum point (4 marks)
- b) Work out the coordinate of the points of inflection of $f(x)$ (5 marks)
- c) Discuss the concavity of $f(x)$ (4 marks)
- d) Sketch the graph of $f(x)$ (3 marks)

QUESTION THREE (20 MARKS)

Determine $\frac{dy}{dx}$

- $5y^2 + \sin y = x^2$ (4 marks)
- $y = \ln\left(\frac{x^2 \sin x}{\sqrt{1-x}}\right)$ (4 marks)
- $ye^x = \sinh(xy)$ (4 marks)
- $y = \tan^{-1}(2t+1)$, $x = e^{-t^2}$ (4 marks)
- $y = 3^{\cos x}(x^x)$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) If $y = \sqrt{x}$ estimate the value of $\sqrt{4.1}$ (5 marks)
- b) A spherical balloon is blown up so that its volume increases at a constant rate of $2 \text{ cm}^3 / \text{s}$. Determine the rate of increase of the radius when the volume of the balloon is 50 cm^3 (7 marks)
- c) Determine $\frac{d^2y}{dx^2}$ for the function $x = 2\cos t - \cos 2t$; $y = 2\sin t - \sin 2t$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Determine;

i) $\lim_{x \rightarrow y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ (5 marks)

ii) $\lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ (10 marks)

b) Given that $f(x) = \cosh^{-1} x$. Calculate $f'(x)$ (5 marks)