MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 305: COMPLEX ANALYSIS I

DATE: 10/12/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the meaning of the following terms

i. harmonic function $u(x, y)$, (3 marks)

ii. harmonic conjugate of a function $u(x, y)$. (3 marks)

b) Determine the fixed points of the bilinear transformation$w = \frac{2z-i}{3z-1}$. (4 marks)

c) Consider the function$f(z) = \frac{z+1}{2z-1}$. Compute the derivative $f'(z)$ hence or otherwise determine the value of $f'(z)$ at the point$z = -3 + i$. (5 marks)

d) Calculate the residues of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at all of its poles. (5 marks)

e) Show that for any $z_1, z_2 \in \mathbb{C}$ then $2\sin z_1 \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2)$. (5 marks)

f) Evaluate $\oint \frac{dz}{z-1}$ over a closed curve C where $C: |z| = 2$ (5 marks)

QUESTION TWO (20 MARKS)

a) Show that $\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4$ (5 marks)

b) State the Cauchy-Riemann equations hence or otherwise determine whether the function $f(z) = z^2\bar{z}$ is analytic or not. (5 marks)

c) Verify that the function $u(x, y) = x^2 - y^2 - y$ is harmonic hence or otherwise determine the harmonic conjugate of u. (5 marks)

d) Given the function $f(z) = \frac{e^z}{(z-2)^2}$, expand $f(z)$ in a Laurent series (5 marks)

QUESTION THREE (20 MARKS)

a) Determine and sketch the images of the lines $x = 3$ and $y = 4$ under the transformation $w = z^2$. (5 marks)

b) Let e^z denote the complex exponential function. Prove that $e^{z_1}e^{z_2} = e^{z_1+z_2}$ for any complex numbers $z_1, z_2 \in \mathbb{C}$ (5 marks)

c) Express $-1+i$ in the form $re^{i\theta}$. Find $(-1+i)^{-8}$ i.e. $\frac{1}{(-1+i)^8}$ as a complex number in standard form. (5 marks)

d) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path $y = x^2$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Using examples, explain the meaning of the following terms

   i. an *injective (or 1-1)* transformation, (2 marks)

   ii. a *bilinear* transformation. (2 marks)

b) Determine the square roots of the number $z = (\sqrt{3} + i)$. (3 marks)

c) Determine the transformation equations for the complex function $w = f(z) = \frac{1}{z^2}$ in polar form. (4 marks)

d) Obtain the Laurent series expansion for $f(z) = \frac{1}{1-z}$ for $|z| < 1$ (4 marks)

e) Determine the bilinear transformation which maps the points $z_1 = 2, z_2 = -i, z_3 = 0$ on the z-plane onto the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively on the w-plane. (5 marks)

QUESTION FIVE (20 MARKS)

a) Express the complex number in its standard form (rectangular form) (4 marks)

$$z = (5 - 2i)\left(\frac{1}{3 - i}\right)$$

b) Determine all the complex numbers w such that $w^3 = -8i$ (4 marks)

c) Evaluate $\int_C \overline{z}\, dz$ where C is the union of the line joining $z = 0$ to $z = 2i$ and the line joining $z = 2i$ to $z = 4 + 2i$. (6 marks)

d) Given that $f(z) = \frac{1}{z^2+1}$ find its residue and evaluate $\oint_c \frac{1}{z^2+1} dz$ where $c: |z - i| = 1$

(6 marks)