



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

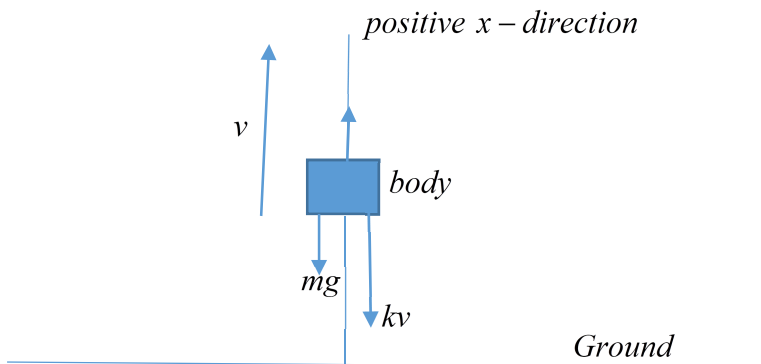
SMA335: ODE 1

Time 2hours

Attempt question *one (compulsory)* and any other *two questions*.

Question 1(30marks)

- a) Determine the differential equation associated with the primitive $y = Ax^2 + Bx + C$ (4marks)
- b) Determine the solution to the differential equation $\frac{dy}{dx} = \frac{xy^2 + x}{yx^2 + y}$ (4marks)
- c) A body of mass m is thrown vertically into the air with an initial velocity v_0 . If the body encounters an air resistance proportional to its velocity, calculate
- i.) The equation of motion in the coordinate system as below (2marks)



- ii.) An expression for the velocity of the body at any time t (3marks)
- iii.) The time at which the body reaches its maximum height (4marks)
- d) Solve the equations;
- i.) $(D - 2)^2 y = 4e^{2x}$ (4marks)
- ii.) $(D^2 + 9)y = \sin 5x$ (4marks)

- e) Solve the Bernoulli's equation $\frac{dy}{dx} - \frac{3}{x}y = x^4 y^{\frac{1}{3}}$ (5marks)

Question 2(20marks)

- a) Define an n^{th} order linear differential equation with variable coefficients (2marks)
- b) Solve the differential equations;
- i.) $2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = x^2$ (8marks)
- ii.) $[D^4 + 2D^3 + D^2]y = \cos 3x$ (10marks)

Question 3(20mks)

- a) Consider the equation $(3x + 2y - 5)dx + (2x + 3y - 5)dy = 0$
- i.) Show that the equation is homogeneous in x and y only (3marks)
- ii.) Solve the differential equation (7marks)
- b) Solve the equations
- i.) $(D - 1)(D + 3)(D - 5)y = \cosh x$ (6marks)
- ii.) $\frac{dQ}{dt} + \frac{2}{10 + 2t}Q = 4$ (4marks)

Question 4(20mks)

- a) Determine the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$. Sketch the orthogonal trajectories. (7marks)
- b) Solve the differential equations;
- i.) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = x^4$ (7marks)
- ii.) $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$ (6marks)

Question 5(20mks)

- a.) Consider the differential equation $(4x + 3y^2)dx + 2xydy = 0$;
- i.) Show that this equation is not exact. (4marks)
- ii.) Solve the equation. (6marks)
- b.) 50 litres tank initially contains of 10 litres of fresh water. At $t = 0$, a brine solution containing 1 gram of salt per gallon is poured into the tank at the rate of 4 litres/min, while the well-stirred mixture leaves the tank at the same rate of 2 litres/min. Determine;
- i.) The amount of time required for overflow to occur; (4marks)
- ii.) The amount of salt in the tank at the moment of overflow (6marks)